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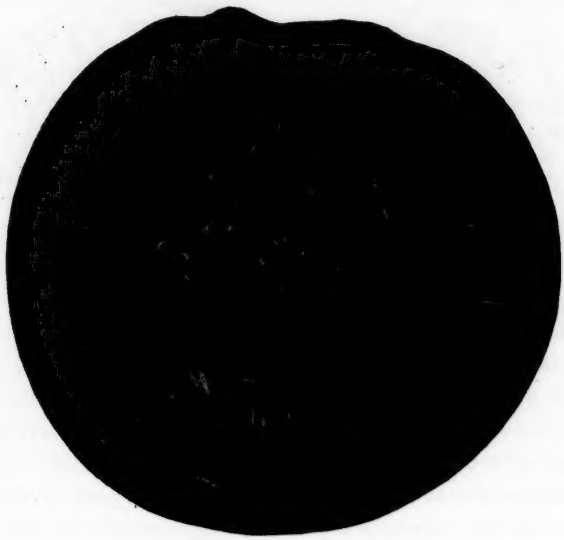
THE MONIST

APOLLONIUS OF TYANA.

A REFORMER of Greek religion from within, whose activity may have coincided with the first emergence of the Christian



APOLLONIUS OF TYANA.
Artistic restoration of the contorniate reproduced on page 162.
(Specially made for *The Monist*.[1])

propaganda from Judæa, is undoubtedly an interesting historical figure. And both in ancient and in modern times Apollonius of

[1] The hand which appears in the original has been omitted because of its inartistic effect.

Tyana has been made the subject of parallels which were probably never thought of by the author of his extant Life. The first of these parallels was by Hierocles, Proconsul of Bithynia under Diocletian; in which the attempt seems to have been made to show that the marvels attributed to Apollonius were better authenticated than those attributed to Christ. We do not possess this work itself; but we have the reply of Eusebius, Bishop of Cæsarea and ecclesiastical historian, written after the triumph of the new faith. The most elaborate modern parallel is that of F. C. Baur, first published in 1832.[1] Baur here attempts to show, not only that there are resemblances between the Life of Apollonius by Philostratus and the Gospels, but that Philostratus deliberately modelled his

APOLLONIUS OF TYANA.
A contorniate, and unquestionably authentic.[2]

hero on the type set forth by the Evangelists. Though he was followed in this view by Zeller, it is now generally rejected; so that there is no need to enter into controversy on the subject. It remains, however, none the less interesting to try to determine the character of the reforming activity of Apollonius himself. Was his predominant aim to conduct the world along the path of intensified supernaturalism, or was it to promote the growth of a more rational

[1] Republished by Zeller with two essays on related subjects under the general title, *Drei Abhandlungen zur Geschichte der alten Philosophie und ihres Verhältnisses zum Christenthum*, Leipzig, 1876.

[2] The medal shows on the obverse the philosopher's head, and on the reverse a victor on a quadriga with a palm in one hand and a whip in the other. The wrong spelling of uncommon words (in the present case TEANEYS for Tyaneus) is not unfrequent in numismatics. Reproduced from Visconti, *Iconogr.*, pl. 17, 4, fig. 115.

and ethical religion so far as this was possible without breaking with the past?

The materials for judging are contained in the Life of Apollonius written by Philostratus early in the third century, and in the extant letters ascribed to him, some of which Philostratus evidently knew. Whether any of these are genuine, it is impossible to be certain; and in any case the biography of Philostratus is clearly a romance. For the composition of it, the writer professes to have used the memoirs of Damis, a disciple of Apollonius; but he tells us that, as these were wanting in literary form, he has freely worked them up. Baur argues that the introduction of "Damis the Assyrian" is simply a literary device. The obvious anachronism by which Philostratus represents the Babylon visited by Apollonius as identical with the Babylon of Herodotus, he also holds to be intentional. It is not, he thinks, put before the reader for serious belief, but only to bring out the ideal attitude of a Greek philosopher confronted with Oriental ostentation. There is much to be said for this view. Philostratus, who was an accomplished man of letters, has nowhere the air of disclaiming credit for the skill of presentation shown in his narrative, while occasionally he disclaims belief in the stories narrated. He was, besides, an original art-critic, as is evident from the descriptions of real or imaginary pictures in another of his works; and he puts into the mouth of Apollonius æsthetic theories which he can scarcely have meant us to believe were not his own. He did not, of course, for a moment suppose that he was drawing up the documents of a new religion, and hence had no motive for concealing his methods. It was only necessary that they should not be obtruded. We have before us a highly mature work of literary art by an individual author who comes forward in his own name. If we cannot be sure in detail about the facts at the ground of the romance, we are saved from the labor of trying to extricate them from stratum on stratum of superimposed redactions. We know at least what type of reformer Philostratus conceived Apollonius to have been.

That Apollonius was a real person born at Tyana, there is no reason to doubt; nor is there any uncertainty about the general

character of his life and teaching. He was in manner of life a Neo-Pythagorean ascetic, and taught what would now be described as a spiritualistic philosophy. The one mode of reforming activity ascribed to him with absolute consistency is a vigorous campaign against animal sacrifices. Superhuman powers, especially those of prophetic insight and of clairvoyance, were attributed to him by common report. Dio Cassius,[1] as well as Philostratus, relates that he saw in a vision the slaying of Domitian. The fact that he had a quarrel with a Stoic philosopher named Euphrates, who is known as a historical personage,[2] is clear, though its causes can only be conjectured from the account of Philostratus. For the rest, there is no ground for supposing that Philostratus deviated in the general spirit of his representation from the authentic type of his hero; and he must have had sources of information open to him for the details, with whatever freedom he may have treated them. Other Lives of Apollonius, now lost, are known to have existed.

In the "Epistles of Apollonius," some of which, as has been mentioned, Philostratus had before him, the type is already individualised. A few points from these may be given as a preliminary to the more detailed biographical account which will follow.[3] The style of the most of them, it may be observed, is of the laconic brevity attributed by Philostratus to all the genuine letters of Apollonius. Two on the subject of sacrifices, addressed to the sacerdotal bodies at Olympia and at Delphi, may be quoted in full. "The gods need not sacrifices. What then might one gratify them by doing? By obtaining wisdom, as I think, and by benefiting worthy men to the extent of one's power. These things are dear to the gods; those are of the godless."[4] "Priests defile altars with blood; then some wonder whence cities are unfortunate, while

[1] Lxvii. 18. See Baur, *Apollonius von Tyana und Christus* (*Drei Abhandlungen, etc.*, ed. Zeller, pp. 110–111).

[2] A laudatory reference to him in the Epistles of the younger Pliny (i. 10) is quoted by Baur, *loc. cit.*, p. 153*n*.

[3] The Epistles of Apollonius and the reply of Eusebius to Hierocles are appended to Kayser's edition of Philostratus, Vol. I. For Philostratus himself I have used Westermann's edition.

[4] Ep 26: τοῖς ἐν Ὀλυμπίᾳ Θεηκόροις.

they do ill in great things. Oh folly! Heraclitus was wise, but not even he persuaded the Ephesians not to wash out mud with mud."[1] The contrariety dwelt on between virtue and riches[2] may be passed over as a well-known philosophic commonplace; but the way in which love of family and country is brought into union with the widest cosmopolitanism seems to offer something distinctive of the philosopher who, having travelled over the known world, is said to have been always pleased when addressed by the name of his birthplace. While it is well, he declares, to think all the earth a fatherland and all men brothers and friends, as being children of God, of one nature; there being the same community of reason and of passions to each and all, barbarian or Greek: yet neither men nor even irrational animals can lose the memory of their home and native seat or find anything to replace it.[3] Men need cities next after the gods; and after the gods cities are to be honored and their interests to be placed foremost by every rational being.[4] While he accepts (or is made to accept) as an honor the charge that his Pythagorean philosophy seeks to attain insight into the future by revelations from the gods—only given, as he contends, to those who are pure in life—he also claims for the Pythagoreans, as Iamblichus did afterwards, the idea of a demonstrable religion.[5] Against the credulity of the time, we find the reproof addressed to the Milesians that, while Thales is called their father, they in their folly accuse a philosopher who predicts an earthquake of causing it.[6] A distinctive point again is the protest against the exaggerated cynical strain in Stoicism. In an imaginary dialogue, Euphrates is made to reproach Apollonius with relieving pains and sufferings of the body (which are no evil, according to the rigor-

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[1] Ep. 27: *τοῖς ἐν Δελφοῖς ἱερεῦσιν.*

[2] See especially Ep. 35: *Ἀρετὴ καὶ χρήματα παρ' ἡμῖν ἀλλήλοις ἐναντιώτατα, μειούμενον γὰρ τὸ ἕτερον αὔξει τὸ ἕτερον, αὐξανόμενον δὲ μειοῖ. πῶς οὖν δυνατὸν ἀμφότερα περὶ τὸν αὐτὸν γενέσθαι; πλὴν εἰ μὴ τῷ τῶν ἀνοήτων λόγῳ, παρ' οἷς καὶ ὁ πλοῦτος ἀρετή.*

[3] Ep. 44.

[4] Ep. 11.

[5] Ep. 52. Among the things received from a Pythagorean teacher, Apollonius mentions, besides mathematical and medical science, *γνῶσιν θεῶν, οὐ δόξαν, εἰδῆσιν δαιμόνων, οὐχὶ πίστιν.*

[6] Ep. 68.

ists). His answer is that the same charge might be brought against the god of healing.[1] Of actual miracles nothing is said; and one who could utter the fine gnomic saying, "To lie is unfree (characteristic of the unfree), truth is noble (characteristic of a noble nature)," cannot well[2] have been a counterfeiter of miraculous powers. A piece of practical advice that may be regarded as a refinement upon this occurs in a letter to a sophist on literary composition: "Since the absolutely best mode of speech is hard to determine, speak in your own character rather than try to imitate what is best—or what you suppose to be best—if you have it not by nature."[3]

One letter obviously different in style from the others[4] is nevertheless interesting as bearing the mark of the period though not of the individual ideas of Apollonius. At the end there is an expression of Stoic pantheism, which, in the transitional phase of the time, was often presented in fusion or confusion with Platonism. Everything done or suffered in appearance by the individual is to be referred to the one first essence (*πρώτη οὐσία*)[5] as its cause, both active and passive. The teaching of Apollonius himself, so far as we can judge, though not without Stoic elements, laid stress rather on the transcendence of the supreme divinity. In the earlier part of the letter, what is supposed to be the Platonic or Pythagorean doctrine of immortality is asserted. Death and birth alike are only appearance. There is alternation between the visible and tangible of nature (*φύσεως*) and the invisible and intangible of essence (*οὐσίας*), but in reality nothing is created or destroyed. The process is conceived as taking place by condensation and rarefaction of matter; the former being the phenomenon of birth and growth, the latter of death. As may be seen, there is here no strictly defined immateriality of the soul, which is either identified

[1] Ep. 8: *τοῦτό που καὶ πρὸς τὸν Ἀσκληπιὸν κοινὸν τὸ ἔγκλημα.*

[2] Ep. 83: *ψεύδεσθαι ἀνελεύθερον, ἀλήθεια γενναῖον.* This may have been a repudiation of the *γενναῖον ψεῦδος* permitted by Plato on occasion to his philosophic guardians of the State.

[3] Ep. 19. [4] Ep. 58.

[5] *ἡ δὴ μόνη ποιεῖ τε καὶ πάσχει πᾶσι γινομένη πάντα διὰ πάντων θεὸς ἀίδιος, ὀνόμασι καὶ προσώποις ἀφαιρουμένη τὸ ἴδιον ἀδικουμένη τε.*

with or very imperfectly discriminated from a fiery or ethereal influx such as the Stoics took to be the basis of life and thought. There seems to be nothing here specially characteristic of Apollonius; but it is clear that in the speculation of the time the Platonic metaphysic was in danger of being swamped in a kind of eclectic animism. The metaphysical advance to a definitely formulated immaterialism as regards the soul itself had to wait for the Neo-Platonic development. Neo-Platonism was in some respects a return from a religious to a more scientific interest in Plato; whose critique of materialism had not hitherto been carried through so rigorously in point of expression that the idea of incorporeal mind and soul could not again be lost.

With this later development we are not at present concerned; and in the teaching of Apollonius himself, as presented by Philostratus, there is abundant interest on the side both of thought and of practice. For the phase to which he belongs, if unoriginal metaphysically, was in other ways marked by advances that proved the still enduring vitality of the ancient culture. It was not indeed by intrinsic decay that that culture disappeared, but by the invasion of alien forces. In the third century it still seemed possible to preserve with modifications the inherited type. The method which commended itself to the minds that were still in the ascendant was that of conservative reform. The imperial monarchy, which no one now dreamed of abolishing, was to be made the centre of institutions as republican as possible in spirit. The ancient religions were to be preserved in some form of union under the ethical direction of philosophy. Oriental cults, severely opposed in the second century, were in the third regarded with more favor if only their underlying community with those of Greece and Rome could be brought into view. The movement found its precursors, both political and religious, in philosophers of the first century; among whom, as we shall see, Philostratus makes it his special aim to assign the place of honor to Apollonius. In more than one respect the philosopher of Tyana was a hero better adapted to the needs of the time than men whose activity had been more characteristic of their own age. Speculative minds were now decisively

turning away from Stoicism and seeking a more transcendental doctrine; and Apollonius had been a Pythagorean. The impracticable character of much of the Stoic resistance to monarchy during the first age of the empire was also recognised; and, while no philosophy would have been listened to that did not repudiate the language of political absolutism, the need was felt of one which laid little stress on the external form of government. This need too was supplied by a Platonising Pythagoreanism which, while it had no more sympathy than the other doctrines with Oriental kingship, assigned a high place among constitutions to a monarchy according to law. To us it is visible that the facts of the situation were making for a formal despotism, a monarchy by divine right, sanctioned by the theocratic Church, now rapidly growing under the surface of ancient life; but this at the time was seen or suspected by few. A still noble civilisation, lowered, as was confessed, in type though extended in range, but accompanied by many advances and possibilities of advance both administrative and spiritual, seemed to thinking men worth preserving against disruptive forces whether from without or from within.

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How far Philostratus was from insisting on the Oriental affinities of his hero may be seen at the opening of his first book where he begins with an apology for them. Some, it appears, refused Apollonius a place among philosophers precisely on the ground that he was said to have put forward his doctrine and discipline as revelations from the gods. Philostratus therefore sets himself to show that, in spite of all that can be urged on that ground, he was a sane and philosophical cultivator of true wisdom as understood among the Greeks. Earlier philosophers also were believed to have been enlightened by divine revelations; and not only Pythagoras himself, but Democritus and Plato and others, had frequented Eastern and Egyptian sages and priests: yet they were not suspected of "magic." His "dæmonic sign" is not brought as an accusation against Socrates. Anaxagoras made meteorological predictions; and these are looked upon as instances of his wisdom. Why then should similar predictions of the future

by Apollonius be ascribed to magical arts? Since, however, he is decried as a magician, and is not generally known in his true character, I have tried, says Philostratus, to bring together the facts from all accessible sources.

The memoirs of Damis, the disciple and companion of Apollonius, he proceeds to explain, were made known to the Empress Julia Domna (the wife of Septimius Severus) by a relation of Damis, and were committed by her to Philostratus, who was a member of her literary circle.[1] Damis, being an Assyrian by birth, was not a skilled writer;[2] but Philostratus has put into shape the materials supplied by him. These, we are to suppose, furnish the groundwork of the narrative.

The story begins with some legends about the birth of Apollonius, agreeably and romantically told. Before his birth he was announced to his mother by "Proteus, the Egyptian god," as an incarnation of himself. "The country people say that he was a son of Zeus, but he calls himself the son of Apollonius."[3] At the age of fourteen he was taken by his father to be instructed by a distinguished rhetorician at Tarsus. Disliking the luxury of the city, he was permitted to migrate to the neighboring Ægæ, where there was a temple of Asclepius. He gave attention to all the philosophies, but attached himself to that of Pythagoras. His Pythagorean teacher did not live according to the principles he taught; but Apollonius, while not ceasing to love his preceptor, aimed at practising the Pythagorean life in all its austerity. Beginning, as he said, like physicians, with discipline of the body, he gave up animal food, both as impure and as coarsening to the intellect. Wine also he gave up, not indeed as impure, but because it makes turbid the æther in the soul. He wore linen garments, rejecting those made from the skins or clothing of animals; went barefoot; let his hair grow long; and took up his abode in the temple. There, Philostrates relates, the god used to appear in person. Apollonius, with his approval, blames the offering beforehand of costly sacrifices, which seem to him to be in intention

[1] i. 3. [2] i. 19. [3] i. 6.

bribes; and bids the priest dismiss a wealthy suppliant, who is a wrongdoer, with his gifts. The gods, he observes, are most just, and will not consent to be bought off in this way. To another evil-minded suitor, he declines the office of mediator, telling him that the gods welcome the good without intermediaries.[1] When he had come of age, he returned to Tyana, having made the temple at Ægæ, says his biographer, a Lyceum and an Academy; for it resounded with all philosophy.

At home, he reformed a debauched elder brother; and when he received his patrimony, distributed most of it among his poor relations, reserving only a small portion for himself. Going beyond the famous precept of Pythagoras, that a man should be faithful to his wife, he resolved on a life of chaste celibacy, and kept his resolution even in youth. According to the Pythagorean rule, he submitted to the probation of five years' silence. During this time, which he passed partly in Pamphylia and partly in Cilicia, he was able to calm factions about games by mere signs. This, says Philostratus, was not so difficult; for people who quarrel about dancers and horses are easily made ashamed of themselves. It was less easy to quell a tumult caused by a famine. This Apollonius did at Aspendus in Pamphylia, where the people were going to burn the prefect, though he had taken refuge by a statue of the Emperor. And at that time, which was in the reign of Tiberius, the Emperor's statues were more terrible and more inviolable than those of the Olympian Zeus. The prefect, on being questioned by signs, protested his innocence, and accused certain powerful citizens, who were refusing to sell corn and keeping it back to export at a profit. To them Apollonius addressed a note threatening expulsion from Earth, who is the mother of all, for she is just, but whom they, being unjust, have made the mother of themselves alone. In fear of this threat they yielded and filled the market-place with corn.

Having completed his probation, Apollonius visited the great

[1] i. 12 (1): "*ξύστησόν με*" *ἔφη* "*τῷ θεῷ.*" *ὁ δ' ὑπολαβών* "*καὶ τί σοι δεῖ τοῦ ξυστήσοντος*"; *εἶπεν* "*εἰ χρηστὸς εἶ; τοὺς γὰρ σπουδαίους οἱ θεοὶ καὶ ἄνευ τῶν προξενούντων ἀσπάζονται.*"

Antioch. He found the people there not only wanting in mental culture, but luxurious and effeminate; and, to judge from the report of Philostratus, seems to have liked that "cradle of the Church" no better than Julian did afterwards. Philostratus here excuses himself for relating myths connected with the temple of Apollo Daphnæus. His purpose, he remarks, is not to mythologise,[1] but to explain how Apollonius came to utter the wish that the god would turn the "semi-barbarous and uncultivated" inhabitants, with their want of all seriousness, into trees, so that thus they might give forth some sound worth listening to. Visiting the temples, he sought to bring back the Hellenic rites to their primitive form: when the rites were alien, he tried to discover their original meaning and to get them corrected in accordance with it. His mode of exposition was not disputatious but magisterial, and this at least gave him some influence with the men of Antioch.[2]

From Antioch he set out with two attendants to visit the Brahmans of India, and, in the course of his journey, the Magi of Babylon and Susa. At Nineveh, Damis, a native of the place, asks and obtains leave to become his companion. Among the Arabians, Apollonius acquires the knowledge they have of the speech of animals. Proceeding on his journey, he encounters a satrap of King Vardanes, the "Mede," who has lately recovered the empire, and whose officials are known as the King's "Eyes" and "Ears." Still, as in the days of Aristophanes, these Oriental titles seem adapted to produce an effect of the grotesque and to form part of the traditional picture of Western Asiatic despotism. We need not try to refer the whole account to the age of Apollonius, though chronologically the Parthian king Bardanes corresponds to the approximate date. The general representation is sufficiently conformable to the revival of the Persian monarchy under the Sassanidæ in the time of Philostratus himself, decked out with circumstances from the historical records of the ancient empire. The narrative is obviously written with a view to contrasting the sim-

[1] i. 16 (2): *οὐχ ὑπὲρ μυθολογίας ταῦτα.*

[2] i. 17 (2): *καὶ ἐπέστρεφεν ἐς ἑαυτὸν ἀνθρώπους ἀμουσοτάτους.*

plicity and independence of the philosopher with the combination of despotism, luxury, and elaborate mechanical art that had distinguished the old civilisations of those regions. Nor is the conception, taken broadly, untrue. The difference of attitude here and in the description of the Indian journey which follows is notable. The Greeks by the time of Philostratus had accumulated some knowledge of India; and, vague as this must have been, it is evident that they had already detected the profoundly philosophical character of the Indian intellect. Thus we are told nothing of what Apollonius was able to learn from the Magi;[1] whereas in the account of his stay in India there is abundance of philosophical interchange of thought. A relatively high but unspeculative religion such as Zoroastrianism or Judaism seems never to have appealed to the Greek mind as did even merely general reports on the tenets of the Brahmans and afterwards of the Buddhists.

Among the decorations of the royal palace at Babylon, we are told, figures of Greek legend were to be seen, Orpheus appearing frequently. Perhaps it is his tiara and his Oriental dress that they are pleased with there: it is not the charm of his music and song. The capture of Athens was represented, and the Persian victory at Thermopylæ, "and things still more Medic, rivers diverted from their course, and the bridging of the sea, and how Athos was cut through."[2] Apollonius refuses to do obeisance to the golden image of the King. The King, who knows him already by repute, is pleased to hear of his coming and sends for him. Being about to sacrifice a white horse to the Sun, he asks Apollonius to accompany him, but the Pythagorean philosopher replies: "You, O King, sacrifice in your own manner, and give me leave to sacrifice in mine." Then, having thrown frankincense on the flame, and uttered a prayer to the god, he departs, so as to have no share in an offering of blood.[3] When the King invites him to join in hunting the animals which the barbarians preserve in their parks or "paradises," he reminds him that he could not even be

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[1] Questioned by Damis (i. 26), he says that they are *σοφοὶ μέν, ἀλλ' οὐ πάντα.*

[2] i. 25.

[3] i. 31.

present at his sacrifices, and expresses disapproval of the pleasure taken in the hunting of wild animals kept for sport.[1] In accordance with the general spirit of the picture, he is represented as neither dazzled by the regal magnificence nor impressed by material marvels such as the tunnel under the Euphrates and the walls of Ecbatana. The King, when he takes leave, provides him with the means of continuing his journey to India; and Apollonius describes him to his companions as an excellent man and worthy of a better fate than to rule over barbarians.

Damis says that in crossing the Indian Caucasus he saw the fetters of Prometheus hanging from the rocks, though it was not easy to tell of what material they were composed. Apollonius frightened off a hobgoblin appearing by moonlight. Then, after these and other strange stories, there follows a remarkable disquisition on the inwardness of the divine.[2] Apollonius questions Damis about the effect on his mind of ascending so high a mountain-range. Damis thinks that he ought to be wiser, passing over such a lofty and trackless spot: "For," said he, looking up at the summit, "you hear from our guide that the barbarians make it to be the house of the gods." Moreover, sages like Anaxagoras and Thales are said to have contemplated the heavens from just such elevations. "Yet," he confesses, "I, having ascended the loftiest height of all, shall go down no wiser than I was before." "Nor did they," replies Apollonius, "merely by such prospects, which display only bluer skies and larger stars and the sun rising from the night—sights visible also to shepherds and goatherds: but how the divinity cares for the human race, and how it delights in being served by it, and what is virtue and what justice and temperance, neither will Athos show those that climb it nor Olympus admired of the poets, unless the soul see through them, which, if it take hold of them pure and undefiled, darts farther than this Caucasus."

Indian nomads having furnished the wayfarers with palm-wine and honey, Damis thinks Apollonius can have no objection to tast-

[1] i. 38: *καὶ ἄλλως οὐχ ἡδὺ θηρίοις βεβασανισμένοις καὶ παρὰ τὴν φύσιν τὴν ἑαυτῶν δεδουλωμένοις ἐπιτίθεσθαι.*

[2] ii. 5.

ing this wine, as it is not made from the grape. Apollonius proves to him that it is really wine, just as coins of bronze are no less money than coins of silver or gold. Moreover Bacchus, whose mountain of Nysa is close at hand, will not be angry with him for not drinking wine at all; but, if he refuses that which comes from the vine and yet drinks that which is made from dates, the god will be angry and think his gift disdained. And other wine, as well as that from grapes, intoxicates, as may be seen in the case of the Indians who drink it. This, however, has been said only to excuse himself, since he is bound by a vow. To his companions he does not wish to forbid wine, nor even flesh.[1]

They meet a boy riding on an elephant, and Damis wonders at his skill in managing such a huge beast. Apollonius by questioning brings out that the credit is due not so much to the boy's skill as to the self-restraint of the animal. Philostratus goes on himself to discuss the various accounts of the elephant, one of them by the Libyan King Juba. The general conclusion is that elephants are second only to man in practical sagacity.

King Vardanes has sent a letter to the satrap placed over the Indus, requesting him to conduct Apollonius on his way. He supplies him with the means of navigating the river, and gives him a letter to his own king. Here Philostratus takes occasion to compare the Indus with the Nile, expressing scepticism in both cases as to the snow which is said to lie upon the mountains and to augment the stream by melting.[2] At Taxila was the king's palace. The dress of the Indians is of linen and of "byssus," which comes from a plant.

Visiting the temple before the city-walls, the travellers find representations with metallic materials on brazen tablets. These were comparable for expression to the works of the best Greek painters.[3] The defeat of Porus and the clemency of Alexander were among the subjects represented. In a dialogue on painting and imitative art generally,[4] Apollonius draws the attention of Damis to the shapes seen in the clouds, which appear to us like

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[1] ii. 7. [2] ii. 18 (2). [3] ii. 20 (2). [4] ii. 22.

centaurs and other forms of living things, known and unknown. He educes the conclusion that, while such shapes are casual so far as external nature is concerned, there is yet an imitative faculty manifested in our seeing them. This faculty is in us. For man is naturally imitative, even when he has not acquired the power of drawing with the hand; and it is this natural imitative faculty, spontaneously exercising itself, that makes us see such shapes. Nor can the faculty be absent in those who merely view pictures, at least if they are to take pleasure in them. To make possible the pleasure in artistic representations, there must be an active power of supplying something from ourselves. This is shown in the case of paintings in black and white. Draw correctly the features of an Indian in white, and he will appear to the fancy as dark; the color being filled in from past experience. So likewise in viewing the picture by Timomachus of the madness of Ajax: he who is to regard it with admiration must bring to it some image of Ajax and some notion of the whole sequence of events of which his madness formed part. The figures on brass seen in the temple are to be classed, Apollonius proceeds, not under the head of mere metal-working (χαλκευτική), but as products of some art intermediate between that and painting in the special sense. It is an art, he concludes, most like that of Hephæstus in Homer's description of the shield of Achilles.

The king, whose name is Phraotes, invites Apollonius to stay for three days. Describing the construction of the city as viewed by the company, and in particular the temple of the Sun, Philostratus takes occasion to note the "symbolical manner" in which the statue of the god was fashioned,—a manner, he remarks, common to the sacred art of all the barbarians.[1] The Greek sage admired the modesty in the ordering of the palace as compared with the luxury of Babylon. In conversing with the king, Apollonius finds him to be a true philosopher. Phraotes, having dismissed the interpreter, requests Apollonius in Greek to let him join him at a banquet. On being asked why he puts his invitation in this form,

[1] ii. 24.

he replies that it is because he regards wisdom as more kingly than his own rank.[1] At the banquet Apollonius asks him how he acquired Greek and came to be possessed of philosophy. The king smiling returns: "As the people of old times inquired of those who came to their coasts whether they were robbers, because that mode of life, though hard, was common, so you Greeks seem to inquire of all who come in contact with you whether they are philosophers; so much do you think philosophy, though the divinest thing that can fall to the lot of men, to be the affair of every one. And indeed I hear that most of those who profess philosophy among you are in fact robbers; the reason being that, while you have laws to punish coiners of false money and such people, you have no law for trying those who claim to be philosophers and for excluding pretenders."[2]

He then proceeds to explain that in India there are few professional philosophers, and that these are carefully tested before they are allowed to enter upon the philosophic life. First their ancestors for three generations back must have done nothing disgraceful; this being ascertained from public records. In the next place the candidates, on offering themselves at the age of eighteen, are examined in respect both of their moral and intellectual fitness. The examiners[3] make use especially of the indications of physiognomy. For where philosophy is held in high honor, as in India, it is most necessary that those who profess it should be subjected to every kind of test. Next Phraotes relates how he himself came to receive a philosophical education. His grandfather was king before him; but his father, having been dispossessed during his minority, was sent for refuge to a foreign king. This king, who had a better realm than his own hereditary one, would have adopted him; but he preferred, as he said, not to contend with fortune, and obtained leave to devote himself to philosophy, so that he might bear his ills more easily. He afterwards married the king's daughter, and brought up his son Phraotes to

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[1] ii. 27 (1): *τὸ γὰρ βασιλικώτερον σοφία ἔχει.*

[2] ii. 29.

[3] Described (ii. 30) as *σοφοί τε καὶ φυσικοὶ ἀνδρες.*

follow the philosophic life. To this end, he taught him Greek. The sages in consequence readily received him as a pupil at twelve, though this was earlier than the usual age; for they regard a knowledge of Greek as a preparation for training in philosophy.[1] Lastly, Phraotes relates how he came to be restored to his kingdom. Apollonius then asks him if the sages he has spoken of did not become subject to Alexander and appear before him to expound their physical philosophy. The King replies that Alexander indeed came in contact with some who profess wisdom of a kind, but who are really a race of warriors. The genuine philosophers of India are those who dwell between the Hyphasis and the Ganges, and to their country his expedition did not extend. Had he gone on, he could never have taken their tower, which, without preparation, they are able to defend by superhuman means.[2]

The next day at dawn the King comes to the chamber of Apollonius and rallies him on his water-drinking. Those who do not drink wine, he says, do not sleep well. Apollonius replying that they sleep more quietly than those who go to bed drunk, the King protests against the sophistry, and explains that his meaning was that those who drink wine in moderation sleep better than those who drink none at all. This leads to an argument in form. Apollonius contends that even moderate wine-drinkers, while not excited to hallucination, are yet liable to be affected by pleasing illusions, and that these too are troubling to the soul and sometimes prevent sleep. They that drink no wine at all remain always equable, neither elated by good fortune nor dejected by bad. Moreover, it is only to the soul untroubled with wine that true divinations come in dreams. Phraotes, having heard the argument, asks Apollonius if he will make him one of his company; but he puts the question by with the remark that it is good for kings to be conversant moderately with philosophy, but that a too exact and overstrained devotion to it would seem unbecoming and pedantic in their station.[3]

At the end of the visit, Phraotes sends Apollonius and his

[1] ii. 31. Here and later the question occurs, Is it possible that anything was known or conjectured as to the affinity between Greek and Sanskrit?

[2] ii. 33. [3] ii. 37.

companions on their journey with new provisions and a letter to Iarchas, "the eldest of the sages." They arrive at the plain in which Porus is said to have fought with Alexander. Beyond the Hyphasis they come upon thirty altars inscribed by the Macedonian conqueror to his father Ammon, his brother Hercules, and the other gods. There is also a stele, they record, marking the place where the expedition stopped. This, Philostratus conjectures, was erected not by Alexander himself but by those beyond the Hyphasis, pluming themselves on his not having been able to go further.

The journey to the remoter regions is accompanied by more and more marvels, zoological and other, which however are related not without occasional touches of scepticism. At last we reach the Tower of the Sages; whom the Indians fear more than the King, because the King himself has to consult them about everything that is to be said or done.[1] A young man sent to meet Apollonius addresses him in Greek; at which the travellers are not surprised, since all in the neighboring village speak Greek. He brings a message from the sages inviting—or rather commanding[2]—him to come. In the form of expression Apollonius recognises something Pythagorean.

Traces were still apparent of the rout of Bacchus, who with Hercules had once made an unsuccessful assault on the tower. The images of the gods were like the most ancient of those among the Greeks, and the rites observed were Hellenic. Apollonius himself, says Philostratus, has described the Brahmans. "They dwell upon the earth and not on it, and are fortified without walls, and possess nothing save the possessions of all men."[3] Out of this the biographer, on the authority of Damis, constructs an account of the Brahmans according to which they raise themselves in the air when they choose—not for the sake of vainglory, but to be nearer the Sun-god, to whom they pray—and are furnished with everything as a spontaneous gift of the earth. Perhaps the con-

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[1] iii. 10 (2). [2] iii. 12: *κελεύουσι γὰρ αὐτοί.*

[3] iii. 15 (1): *εἶδον Ἰνδοὺς Βραχμᾶνας οἰκοῦντας ἐπὶ τῆς γῆς κοὐκ ἐπ' αὐτῆς, καὶ ἀτειχίστως τετειχισμένους, καὶ οὐδὲν κεκτημένους ἢ τὰ πάντων.*

jecture is permissible that we have here some real saying of Apollonius misapprehended by a disciple.

Iarchas addresses Apollonius in Greek, and gives proof that he has the minutest knowledge of his whole history. Going in choral procession to the temple, the Brahmans chanted an ode like the pæan of Sophocles which is sung at Athens to Asclepius. After the service of the gods, in which Apollonius had taken part, he asks Iarchas if the Brahmans, knowing all things else, "know themselves." Iarchas instantly replies that it is in consequence of knowing first themselves that they know all things. "What then," asks Apollonius, "do you think yourselves to be?" "Gods," he answered; and, being asked why, "Because," he said, "we are good men." In answer to the question, what they think about the soul, he replied, "We hold the doctrine that Pythagoras handed down to you, and we to the Egyptians." Apollonius then asks whether, as Pythagoras declared himself to have been Euphorbus, Iarchas too can say that before he came into this body he was one of the Trojans or Achæans or some one else. Iarchas thereupon makes the observation that the Greeks are too much pre-occupied with the Trojan war and its heroes, and neglect the greater number of more divine men whom their own land and the land of the Egyptians and that of the Indians bore. Then he says that he too will declare who he was. He proceeds to relate the history of an ancient Indian king named Ganges, who was the son of the River-god. In that he founded cities instead of destroying them, and drove back an invasion of the Scythians from beyond Caucasus instead of bringing the yoke of slavery upon another city,[1] this king was superior to Achilles. More of his deeds would Iarchas record if he did not shrink from praising himself. For he, at the age of four, revealed his identity by discovering seven swords embedded in the earth by King Ganges, and now sought for to fulfil a command of the gods. He then asks Apollonius if he also knows who he was formerly. Apollonius replies that he does, but that his position was an inglorious one. He was the steersman of an Egyptian ship.

[1] iii. 20 (3): *καὶ ταῦθ' ὑπὲρ γυναικός, ἣν εἰκὸς μηδ' ἀκουσαν ἀνηρπάσθαι.*

In that capacity, however, he once performed a just deed in refusing to betray his ship to Phœnician pirates. This leads to a question about the use of the word "justice," afterwards more fully discussed when Apollonius visits Egypt. Iarchas raises the problem by his criticism that the Greeks seem to think the absence of injustice equivalent to justice, whereas a positive conception is needed.[1]

During the visit of Apollonius to the dwelling of the sages, the King entered. He was not a philosopher like Phraotes, but came arrayed more in the fashion of the Medes, and full of pomp. While the sages themselves ate sparingly, abundance was provided for the King; though it is not lawful for him to partake of the flesh of animals in their presence. He approaches with profound respect the sages who keep their seats. For the repast, self-moving tripods come in, and there are automata to serve as cup-bearers. Apollonius asks Iarchas why he sees precisely eighteen Brahmans present, since eighteen is neither a "square number" nor any other of those that are in repute. Iarchas replies: "Neither are we slaves to number nor number to us."[2] Sometimes they are more, and sometimes fewer, according as there are more or fewer of sufficient wisdom and virtue to be chosen. Then he goes on to blame the Greek democratic mode of appointing to offices by lot, and the fixing of ruling bodies in the Greek cities at a particular number. The king interrupts the conversation by asking questions about the Greeks, of whom, however, he has a mean opinion; imagining, for example, that the Athenians had been enslaved by Xerxes. Apollonius corrects this impression. Xerxes, he maintains, was unfortunate in not having died as well as been defeated at the hands of the Greeks, who in that case would have instituted games in his honor, thinking as they do that it is a praise to themselves to praise those whom they have vanquished. The King explains that he had got his false opinion from the Egyptians, who abuse the Greeks as borrowers of everything from themselves, and as a race of law-

[1] iii. 25.

[2] iii. 30 (2) *οὐθ' ἡμεῖς ἀριθμῷ δουλεύομεν οὔτ' ἀριθμὸς ἡμῖν.*

less cheats. He invites Apollonius to be his guest, but the invitation is declined.

Iarchas and his associates, questioned by Apollonius as to their views on the constitution of the world, reply that they hold it to consist of elements (ἐκ στοιχείων). These are the four elements of water and air and earth and fire, together with ether as the fifth. No element came into being before the others, but all exist together as parts of the living whole. This is at once male and female, and is held in unison by love of itself. The parts of the world are governed by the mind that is in it. As bearing an analogy to this government of the fabric by mind, Iarchas describes a merchant ship such as the Egyptians send to India. In the vessel of the world, the first place is to be assigned to God the begetter of this animated universe (θεῷ γενέτορι τοῦδε τοῦ ζῴου); the next to the gods who preside over the parts. Of such deities, following the poets, we may admit many, of sky and sea and springs and earth and under the earth. The place beneath the earth, however, since they sing of it as an abode of horror and destruction, does not, if it exists, belong properly to the world.[1]

As an illustration of the powers of the sages, some extraordinary cures are related. A woman comes and explains how her son is possessed by a dissembling and lying demon. One of the Brahmans gives her a formula of exorcism addressed to the demon.[2] A cripple, and a blind man, and a man with his hand paralysed are healed, and recipes are given to effect other cures.

According to his report, Damis was himself present at the dialectical discussions. The study of astrology and divination and sacrifices was pursued only by Apollonius with Iarchas. Philostratus mentions works of Apollonius on these subjects; but remarks that in his own opinion astrological prediction, with all such divination, is beyond the scope of human nature: whether any one has attained to it he does not profess to know. The work of Apollonius on Sacrifices is in so many hands, and is so well and characteristically composed, that exposition of it is unnecessary.[3]

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[1] iii. 34, 35. [2] iii. 38. [3] iii. 41 (2).

Since Damis has given an account of a conversation on the strange animals and so forth of India, Philostratus, while declining to commit himself to the truth of the stories, will not wholly pass the subject by.[1] For the rest, the account of the Indian journey ends, as it begins, with enough of the marvellous. Philostratus was on the whole content to put into literary form the travellers' tales he knew; hinting sometimes to the less credulous his uncertainty as to what grains of truth might be found in the more extraordinary of them.

After a stay of four months, Apollonius leaves the Brahmans. A letter is given as from him in which he is made to say that he has received from them the power of going through the sky (*διὰ τοῦ οὐρανοῦ πορεύεσθαι*) and of conversing with them at a distance as if they were present.[2] He and his companions return to the region of the Indus, then put to sea, and sail up the Euphrates to Babylon. Returning to the Roman Empire, they go to Antioch; but, finding it as insolently indifferent as ever to Hellenic culture,[3] they put to sea again at Seleucia, and sail thence to Cyprus. From Cyprus they proceed to Ionia, where Apollonius is held in much honor.

When he came to Ephesus, we are told,[4] even the artisans left their work to follow him. He delivered a discourse to the Ephesians in favor of a voluntary community of goods; teaching by the example of a sparrow that came to call the others to join him in feasting on the corn spilt by a boy carrying a basket. He foresaw a threatening pestilence, but, as they did not heed his warnings, he went to the other parts of Ionia; continuing everywhere his reforming activity and his salutary discourses.[5]

A discourse at Smyrna is given[6] in which he exhorts the Smyrnæans to make themselves an object of pride even more than the

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[1] iii. 45 (1): *καὶ γὰρ κέρδος εἴη μήτε πιστεύειν μήτ' ἀπιστεῖν πᾶσιν.*

[2] iii. 51.

[3] iii. 58: *τῆς 'Αντιοχείας ξυνήθως ὑβριζούσης καὶ μηδὲν τῶν 'Ελληνικῶν ἐσπουδακνίας.*

[4] iv. 1.

[5] iv. 4: *διορθούμενος τὰ παρ' ἑκάστοις καὶ διαλεγόμενος ἀεί τι σωτήριον τοῖς παροῦσιν.*

[6] iv. 7.

beauty of their city. For although it is the fairest of all cities that are under the sun, and possesses the sea, and has the springs of the west wind, yet it is better for it to be crowned with men than with porticoes and paintings and greater abundance of gold. Buildings are seen only in that part of the earth where they are; but good men are seen everywhere and spoken of everywhere, and render the city they have sprung from as wide as the extent of land they penetrate. Cities that are fair externally are indeed like the Phidian image of Zeus at Olympia: but those that have men that reach every part of the world are like the Homeric Zeus, who is suggested to thought in various forms, and as moving through the heaven, and so is a more wonderful piece of work than the seated statue of ivory visible to the eye. Discussing politics with the Smyrnæans, he told them that a rightly ordered city has need of concord in variance.[1] That is to say, each must make it his ambition to be better than the rest in something. The ancient Spartans were wrong in their exclusive devotion to military affairs. Each ought to do what he knows best and can do best. If one gains distinction by becoming a popular leader, another by wisdom, another by amassing wealth for the common good, and so forth, then the whole city will stand firm. This he illustrates by the example of a ship with its division of employment.

The plague having actually fallen upon Ephesus, the Ephesians sent an embassy to Apollonius. He was there on the instant —as Pythagoras was at Thurium and Metapontum at the same time—and stayed the plague by destroying a demon in the guise of an old beggar-man, revealed afterwards as a monstrous beast.[2] He decided on a voyage to Greece, but first visited the tomb of Achilles in the Troad. When they were afterwards sailing the Euboic Sea, Damis questioned him about his visit, and Apollonius recounted his conversation with the shade of the hero, which disappeared with a glimmer at cockcrow.[3] Arriving at the Piræus at the time of the Eleusinian mysteries, he was joined on his way to

[1] iv. 8 (1): *ὁμονοίας στασιαζούσης.* [2] iv. 10.

[3] iv. 16 (6): *ἀπῆλθε ξὺν ἀστραπῇ μετρίᾳ· καὶ γὰρ δὴ καὶ ἀλεκτρυόνες ἠδὴ ᾠδῆς ἤπ-οντο.*

Athens by ten young men who were about to set sail to Ionia to see him. He offers himself for initiation in the mysteries; but the hierophant raises objection to him as an enchanter and as "not pure in respect of divine things." Made aware of the popular disapproval, he changes his tone; but Apollonius now declines initiation till another time; mentioning the name of the successor to the office, who, as he foresees, will initiate him. At Athens, in deference to the devotional spirit of the place, he first discoursed about sacrifices, thus refuting the calumnious and ignorant assertion of the hierophant.[1] A youth who interrupts a discourse of his with inane laughter he finds to be under demoniacal possession. The demon, being commanded to go out of him and to give a sign of his departure, says that he will throw down a statue on his way. This he does to the astonishment of the audience. The youth afterwards followed the philosophic mode of life.[2] Hearing of the frivolities with which the Athenians were now accustomed to celebrate the Dionysia, Apollonius rebuked them by reminding them of the exploits of their ancestors and of their legendary connection with Boreas, the most masculine of the winds. Another abuse which he arrested at Athens was the introduction of gladiatorial exhibitions,—which were going on, Philostratus tells us, at Corinth in his own day.

In a journey to Thessaly, Apollonius visited the tomb of Leonidas, which he all but embraced.[3] When his companions were disputing which was the highest mountain in Greece, he ascended the height where the Spartans had been overwhelmed by the Persian arrows, and said that those who died there for liberty had equalled it to Œta and raised it above many an Olympus. In the account of his residence at Corinth we come upon the original of the story of Keats's *Lamia*.[4] This occurrence, says Philostratus, was already well known, but only in general outline, and as having taken place in central Greece. He has given the details for the first time from the record of Damis. At Olympia, receiving an in-

[1] iv. 19: *τίς γὰρ ἐτ' ὠήθη τὰ δαιμόμια μὴ καθαρὸν εἶναι τὸν φιλοσοφοῦντα, ὅπως οἱ θεοὶ θεραπευτέοι;*

[2] iv. 20. [3] iv. 23: *μονονοὺ περιέβαλεν.* [4] iv. 25.

vitation to Sparta from a Lacedæmonian embassy which he observed to be full of luxury, Apollonius wrote to the ephors on the subject and brought about a restoration of the ancient manners. A conceited youth submitted to him a long panegyric on Zeus. The philosopher asked him if he had ever written in praise of his own father. He replied that he should have liked to do so, but found that he could not do it adequately. "Then," replied Apollonius, irritated as he was apt to be by vulgar pretence,[1] "If you do not think you can fitly praise your father whom you know, do you not see that, in undertaking to praise the father of gods and men and the fashioner of all that is around us and above us, you have entered upon a task beyond human powers?"

One incident of his visit to Sparta may be quoted for the light it throws on his general attitude as a reformer. A young man who was a descendant of Callicratidas, the Spartan admiral at Arginusæ, had an action brought against him because he had adopted a sea-faring life for gain, and because he took no part in public affairs. Apollonius succeeded in convincing him that in this he was derogating both from his ancestral traditions and from those of Sparta. He accordingly gave up his mercantile pursuits, and at the intercession of Apollonius was acquitted by the ephors. Superficially this may seem inconsistent with the discourse at Smyrna, but in reality it is part of the same general ideal. In that ideal, local diversity is included. Thus at Athens, as we shall afterwards find, Apollonius will allow no disrespect towards the sea-faring tradition; whereas here he reminds the descendant of Callicratidas that the Spartans lost their military power when they took to the sea.[2]

The humanitarian tendency which the reforming movement combined with its regard for antique ideals becomes evident when we are told expressly that Apollonius treated the slaves of his companions as a part of his philosophic community.[3] Passing over

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[1] iv. 30 (3): *δυσχεράνας οὖν ὁ Ἀπολλώνιος (τοῦτο δὲ πρὸς τοὺς φορτικοὺς τῶν ἀνθρώπων ἔπασχεν).*

[2] iv. 32. Cf. v. 20.

[3] iv. 34 (2): *κοινὸν δ' ἐκάλει τοὺς θ' ἑταίρους καὶ τοὺς τῶν ἑταίρων δούλους· οὐδὲ γὰρ ἐκείνους παρεώρα.*

some intermediate incidents, we may follow him westward to Rome, where at this time Nero was persecuting philosophy.

The philosophic cloak, says Philostratus, was proceeded against in the law-courts as a disguise of diviners. Not to mention other cases, Musonius, a man second only to Apollonius, was imprisoned on account of his philosophy and came near losing his life. Before Apollonius and his company reach the gates, a certain Philolaus of Citium tries to deter them from proceeding. To Apollonius this seems a divinely-ordained test to separate the stronger disciples from the weaker (whom, however, he does not blame); and, in fact, out of thirty-four, only eight remain with him, the rest making various excuses for their flight at once from Nero and from philosophy. Of those who remained was the young man whom he had rescued from the transformed serpent.

He stigmatises the reigning tyranny as one so grievous that under it men are not permitted to be wise.[1] His discourses being all public, no accusations were made against him for a time. He did not seek out men of position, but welcomed them if they came, and discoursed to them exactly as to the common people.[2] At Corinth a Cynic philosopher named Demetrius had attached himself to him as Antisthenes did to Socrates. This man now came to Rome and brought suspicion on him of responsibility for the violent attacks he himself proceeded to make on Nero. A public protest against luxury delivered on a feast-day in a gymnasium which the Emperor was opening in person led to his expulsion from Rome by Nero's minister Tigellinus, who henceforth kept a close watch on Apollonius. His opportunity came at last when there was an epidemic of colds and the temples were full of people making supplication for the Emperor because he had a sore throat and the "divine voice" was hoarse. Apollonius, bursting with indignation though he was at the folly of the multitude, did not chide any one, but tried to calm a disciple by telling him to "pardon the gods, if they delight in buffoons." This saying being reported to Tigellinus, he had him arrested under the *lex majestatis*. On

[1] iv. 38 (2): *τυραννίδος....καθεστηκυίας οὕτω χαλεπῆς, ὡς μὴ ἐξεῖναι σοφοῖς εἶναι.*
[2] iv. 41.

bringing him to trial, however, he found himself baffled, and, in fear of his superhuman powers, let him go.[1]

An incident at Rome is recorded that was thought to be an illustration of those powers. A maiden who was about to be married had died or appeared to have died, and was being carried to the grave amid the lamentations of all Rome; for she was of a consular family. Apollonius, meeting the funeral procession, commanded them to set down the bier, and, saying something inaudible, restored the maiden to life; who then, like Alcestis brought back by Hercules, returned to her father's house. Whether he detected a spark of vitality that had escaped the notice of the physicians, or renewed the life that was extinct, Philostratus acknowledges to be beyond his own judgment, as it was beyond the judgment of those who were present.[2]

The next voyage of Apollonius was to the region of the Baetis in Spain. Philostratus here tells some anecdotes to illustrate the greater or less civilisation of the surrounding country. When a courier came to Gades to announce the triple victory of Nero at Olympia, the people there understood what was meant; but those of the neighboring cities, who knew nothing about the Greek games, got the notion that the Emperor had been victorious in war and had taken captive certain "Olympians."[3] A tragic actor came to Hispalis. Where the people retained less of the antique barbarism in their manners, they were pleased with tragedy as a new thing; but here the mask, and the lofty elevation of the actor, and the portentous robes, and the resonant voice, terrified them till they fled as from a demon.[4] Apollonius was sought out by the prefect of the province. The subject of their conversations is unknown; but Damis conjectures that they plotted against Nero; for, when the prefect took his leave, the last words of Apollonius were, "Farewell, and remember Vindex." Philostratus reminds the reader that it was Vindex who first stirred up the peoples of the West against the Emperor when he was making his progress

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[1] iv. 44 (4): "*χώρει*," *ἐφη* "*οἱ βούλει· σὺ γὰρ κρείττων ἢ ὑπ' ἐμοῦ ἀρχεσθαι.*"

[2] iv. 45. [3] v. 8. [4] v. 9.

through Achaia; and mentions that he addressed to his soldiers an oration such as one inspired by the noblest philosophy might breathe forth against a tyrant.[1]

Apollonius and his companions proceed by way of Africa to Sicily. Hearing of the flight of Nero and the death of Vindex, Apollonius in an oracular utterance predicts the brevity of the reigns of the next three emperors (Galba, Otho, and Vitellius). Such predictions Apollonius made, his biographer insists, not as an enchanter, but so moved by a divine impulsion as to know what the fates had in store. Enchanters or magicians (οἱ γόητες), "whom," says Philostratus, "I regard as the most wretched of men," proclaim that by juggling artifice and by barbarian sacrifices they can change the purpose of the fates; and many of them, when judicially accused, have confessed that this is the nature of their wisdom. Apollonius, on the other hand, followed the decrees of the fates, and foretold what would come to pass of necessity. So far was he from all juggling that when he saw the automata in India he praised the ingenuity of the contrivances but did not care to learn the details of their mechanism.[2]

At Catana, the story is told that Typho is bound there, and that from him arises the fire of Ætna. Apollonius takes this occasion to lead his disciples to a more "physical" view of volcanic eruptions. He begins with a paradox on the fables of Æsop; that they are to be preferred to those of the poets, in respect of wisdom, precisely because they are not told with such coloring as to give the impression that they are literally true. The didactic purpose in them is thus made obvious; whereas the poets leave it to the intelligence of their readers to discover the truth. He himself relates a story about Æsop and Hermes, told to him as a child by his mother; the point of the story being that the god had suggested to Æsop a line of invention that was at least his own, if it was humble. As for the myth about the contention of giants with gods for the possession of heaven, this is madness to say or to think.[3]

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[1] v. 10 (2): *λόγον....ὃν ἐκ πάνυ γενναίας φιλοσοφίας ἐπὶ τύραννον ἂν τις πνεύσειεν.*

[2] v. 12.

[3] v. 16.

The cause of these outbursts of flame from volcanoes is in reality a mixture of bitumen and sulphur blown upon by subterranean winds in the crevices of the earth.

Revisiting Athens, Apollonius is initiated into the mysteries, as he had foreseen. The winter he spends in visiting the Greek temples. He projects a voyage to Egypt in the spring, and, going down to the Piræus, finds a ship. The owner refuses to let him go on board, because, as he is conveying a cargo of images of the gods, he is afraid to admit sea-faring company, which is usually bad. Apollonius reminds him—since he appears to be an Athenian—that the gods themselves when they went on board the ships and took part with Athens against the barbarians, had no fear of contamination from disorderly sailors. He also censures the traffic in images.[1]

At Rhodes he tells a newly-rich and uneducated youth who is building a fine house and collecting paintings and statues for it that he does not seem to possess the house, but the house to possess him.[2] Coming to Alexandria, he is treated with great reverence. Here an example was seen of his marvellous powers. Twelve men condemned for robbery were being led to execution. He perceived that one of them was innocent, and told the executioners to place this man last; meanwhile prolonging his speech so as to gain time, contrary to his custom of brief utterance. When eight had been decapitated, a horseman rode up with a reprieve for the prisoner on whose behalf Apollonius had interceded; his innocence having since been established.[3]

We are told of a dispute in the temple with an Egyptian priest regarding animal sacrifices, and of a discourse reproving the Alexandrians for the sanguinary quarrels that arose from their devotion to the contests of the hippodrome. At this point of the narrative, Vespasian arrives in Alexandria from Judæa, aiming now at the Empire. The philosophers Dion and Euphrates bid the people rejoice. For, says Philostratus, the last fifty years had been a period of tyrannies so harsh that even the reign of Claudius, though

[1] v. 20. [2] v. 22 (2). [3] v. 24.

he was better than the emperors before and after, had seemed to give no respite.[1] Apollonius was equally glad, but did not care to obtrude himself. Vespasian, however, sought him out, and first set forth to him alone his reasons for seeking the empire; though he had commended to him his fellow-philosophers also as advisers. Apollonius heartily approves of his purpose; and, to his astonishment, tells him that he is destined to rebuild the temple of Jupiter Capitolinus at Rome. He did not yet know that the temple had been burned down; but it was afterwards found that this was the case, and that the conflagration had been manifested to Apollonius sooner than if it had taken place in Egypt.[2]

The day after the private interview with Apollonius, the other philosophers are called in. Vespasian formally explains to them the motives of his action; describing the tyranny to which the Roman world has been subject from the reign of Tiberius, and pointing out that if Vitellius is allowed to rule, Nero will have come to life again. "You have learned how not to govern," said Apollonius, "from those who governed badly: let us now consider how a good ruler ought to act."[3] Euphrates, however, who has become jealous of the special attention paid to Apollonius, makes a long speech in Stoic phraseology; first remarking that it is premature to consider how one is to proceed in a certain course of action before it has been decided whether that is the right course. In the end he approves of the resolution of Vespasian to march against Vitellius, but advises that, if he is victorious, he should restore to the Romans the democratic form of government under which they were most prosperous, and gain for himself the glory of having begun an era of freedom. Dion partly agrees and partly disagrees with the advice of Euphrates. He agrees in particular that Vespasian would have done better to let the Jews separate if they

[1] v. 27. Tacitus also dated the beginning of improvement from the reign of Vespasian, to whose personal example he ascribes some influence in the return from excessive luxury to a simpler mode of living: "Nisi forte rebus cunctis inest quidam velut orbis, ut quem ad modum temporum vices, ita morum vertantur; nec omnia apud priores meliora, sed nostra quoque aetas multa laudis et artium imitanda posteris tulit." (*Ann.* iii. 55.)

[2] v. 30.

[3] v. 32 (3).

chose; political separation being appropriate to the singularity of their manners. Instead of spending his force in bringing them to subjection, and thus doing all that was in his power to preserve the empire for Nero, he ought to have straightway attacked him. At the same time he approves of the enterprise against Vitellius. A democracy, if inferior to an aristocracy (of Platonic type), is to be preferred to tyrannies and oligarchies; but he fears lest the Roman people, tamed as they now are by a series of tyrannies, should find the transition to liberty as unbearable as that from darkness to sudden light. Let Vespasian, however, put the question to the vote, and if the people choose democracy, grant it. In that case he will win fame universal and unparalleled. If, on the other hand, they choose monarchy, who should be emperor but himself? Apollonius demonstrates at length the impracticability of all this. To him personally the form of political government is indifferent, since he lives under the gods; but he does not think that the human flock ought to be left to perish for want of a just and prudent pastor. As one man pre-eminent in virtue, when he becomes ruler in a democracy, makes that polity seem identical with the form of government in which the one best man rules; so the government of one, when it keeps steadily in view the good of the commonwealth, is in effect a democracy.[1] At Vespasian's request Apollonius, premising that the art of government is not a thing that can be taught, goes on to lay down some general maxims for the exercise of kingly power. The king is himself to be ruled by the law. Vespasian personally is advised not to let his sons take for granted that the empire will fall to them as his heirs, but to teach them to regard it as the prize of virtue. He is not to go too fast in repressing the pleasures to which the people have become accustomed: they must be brought to temperance by degrees. Governors of provinces should know the language of the provinces they are sent to govern. The disadvantage of not observing this rule he illustrates from the failure in the administration of justice when he was in the Peloponnese; the Roman governor, who did not know Greek,

[1] v. 35 (4).

being at the mercy of those who had an interest in deceiving him. Euphrates allows that further discussion would be idle, since the course to be taken has already been resolved on; but, with an allusion glancing at Apollonius, gives the future emperor the parting advice to embrace the philosophy that is according to nature, and to have nothing to do with that which professes itself inspired by the gods, liable as such claims are to be the source of deception.[1] Vespasian perceives his animus: and, when Euphrates afterwards hands him an epistle full of requests of presents for himself and his friends, he reads it aloud; thus giving Apollonius the opportunity of retorting on Euphrates by contrasting his readiness to ask for gifts from the emperor with his counsel to establish a democracy.

This, Philostratus tells us, is what he has been able to learn about the origin of the difference between the two philosophers. With Dion, Apollonius was always on good terms, though he thought Dion's philosophy too rhetorical. Euphrates, according to the story, was afterwards in favor under Domitian. When Vespasian as emperor revoked the liberty granted by Nero to Greece, Apollonius did not care to see him again; though he approved of his good administration generally. In connection with the story of the philosopher at Alexandria, a strange tale is recounted of his detecting the soul of King Amasis in a tame lion.[2] He left Alexandria on a journey to Æthiopia, accompanied by ten disciples out of the number that had again gathered round him since the dispersal under the persecution of Nero.[3]

On the borderland between Egypt and Æthiopia a primitive system of barter was practised. This Apollonius praised for its moral superiority over the habits of commercial bargaining among the Greeks.[4] An Egyptian youth named Timasio, who had overcome a temptation similar to that of Hippolytus, guided the company to the celebrated statue of Memnon. Apollonius praises him for his

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[1] v. 37 (1): *φιλοσοφίαν δέ, ὦ βασιλεῦ, τοντὶ γὰρ λοιπὸν προσειρήσεται, τὴν μὲν κατὰ φύσιν ἐπαίνου καὶ ἀσπάζου, νὴν δὲ θεοκλυτεῖν φάσκουσαν παραιτοῦ· καταψευδόμενοι γὰρ τοῦ θείου πολλὰ καὶ ἀνόητα ἡμᾶς ἐπαίρουσιν.*

[2] v. 41. [3] Cf. iv. 37. [4] vi. 2.

continence, and regards him as of more merit than Hippolytus because, while living chastely, he nevertheless does not speak or think of the divinity of Aphrodite otherwise than with respect.[1] He and his companions, still guided by Timasio, arrive among the Gymnosophists, whom they have set out to visit. In consequence of a trick of Euphrates, who has sent his disciple, Thrasybulus of Naucratis, to prepossess them against Apollonius, they put off receiving him for some time. At length, after negotiations through Damis, who detects the trick, they consent to receive him, though still resenting his reported preference of the Indian wisdom to their own.

The eldest and chief of them, who is called Thespesio, delivers a long address, in which he seeks to place the aims of himself and his associates above those of the Indians; advising Apollonius to have no care about automata or wonder-working, but to choose rather the wisdom that goes with toil and simplicity of life. In his peroration, he reminds him of the pictures he must have seen of the Hercules of Prodicus. As in the Choice of Hercules Vice stands on one side luxuriously adorned, and Virtue poorly clad on the other; so let Apollonius think of himself as placed between the alluring wisdom of the Indians and the rugged discipline of the Egyptian Gymnosophists. Apollonius explains that he has not come thus late to make his choice between two philosophies. Surrounded as he was in his youth by the teaching of all the schools, he of his own accord adopted the Pythagorean discipline, in spite of the austerity which from the first it did not conceal.[2] Among the rewards it promises to its votaries is to appear more pleasing to the gods though sacrificing little than do those who pour forth to them the blood of bulls. The doctrine of Plato regarding the soul, divinely taught by him at Athens, Apollonius perceived not to have won general acceptance among the Athenians. He therefore sought out a city or nation in which one person should not say

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[1] vi. 4.

[2] Special stress is laid on the virtue of chastity. The Pythagorean philosophy is represented as addressing the neophyte: *κἂν ἀφροδισίων ἡττηθέντας αἰσθωμαι, βάραθρά ἐστί μοι, καθ' ὧν σοφίας ὀπαδὸς δίκη φέρει τ' αὐτοὺς καὶ ὠθεῖ.* See vi. 11 (5).

one thing and another the opposite, but the same doctrine should be confessed by all. First, accordingly, he looked to the Egyptians; but his teacher told him that the original fathers of this wisdom were the Indians. For the rest, he addresses to the Gymnosophists an apology on behalf of the arts and graces of life and the adornment of temples; pointing out that Apollo does not disdain to clothe his oracles in verse, and that self-moving tripods are introduced by Homer at the banquets of the gods. Nor has any accusation yet been laid in heaven against Hephaestus for corrupting matter by his art. Every art will have a care for ornament; because the very being of arts was invented for the sake of ornament.[1]

The speech of Apollonius made a powerful impression on all, and especially on Nilus, the youngest of the Egyptians. Thespesio, though black, might be seen to blush. This, of course, is a reminiscence of Thrasymachus in the *Republic*, as it is likewise when we are told that he becomes reconciled with Apollonius. Requested by Nilus and Thespesio, Apollonius recounts his adventures. He and his companions are courteously entertained by Nilus at a repast. Nilus desires to become his companion; and, to show that this is no rash impulse on his part, relates his history. His father had sailed to the Indian coast, and had told him what he had heard about the sages in India. Informed by him also that the Gymnosophists were a colony from thence, he gave up his patrimony and joined them. He found them wise indeed, but not like the Indians; and had he not met with Apollonius, he would himself have sailed to India like his father. The eager and ingenuous Nilus also proposes that he shall try to persuade his elders of the inferiority of their wisdom: but this Apollonius discountenances; receiving him on condition that he will not make an attempt which would be of no avail.

Apollonius pays a visit to Thespesio, and asks him to instruct him in the Egyptian wisdom, so that he may communicate it to

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[1] vi. 11 (17): *κόσμου γὰρ ἐπιμελήσεται τέχνη πᾶσα, ὅτι καὶ αὐτὸ τὸ εἶναι τέχνας ὑπὲρ κόσμου εὑρηται.*

others, as he has communicated that which he received from the Indians. Thespesio signifies his readiness to answer questions. Apollonius begins by asking why the Egyptians represent the gods for the most part so absurdly; their sacred images being apparently made in honor of irrational animals rather than of divine beings. Thespesio parries this attack on zoömorphism by a similar question about the anthropomorphism of the Greeks. Did your Phidias and your Praxiteles ascend into heaven and copy the forms of the gods? And if it was not imitation that produced their art, what then could it be? An artist of more wisdom, answers Apollonius, namely, imagination.[1] He who conceives the form of Zeus must see him in his mind's eye accompanied by the heaven and the seasons and the stars: the fashioner of a statue of Athena must think of armies, and of wisdom in counsel, and of the arts, and of how she sprang from the head of Zeus. Thespesio, on his part, contends that the Egyptians display more reverence to the gods in not audaciously trying to realise some conception of their forms, but using only symbol and suggestion. Apollonius replies that there is nothing to call forth reverence in the image of a dog or an ibis or a goat. If, as Thespesio says, that is regarded with more reverence which is only suggested to the mind, then the Egyptians should have had temples and rites indeed, but no images at all; leaving the mental representation entirely to the worshipper. "But you," he says in concluding, "have taken away from the gods both visible beauty and the beauty of suggestion."[2] "There was a certain Athenian named Socrates," is the retort of Thespesio, "an old man of no intelligence like ourselves, who used to swear by the dog and by the goose and by the plane-tree." "Not that he thought them gods," returns Apollonius, "but so that he might not swear by the gods."

Thespesio, as if changing the subject, inquires about the scourging of boys at Sparta. Do the Greeks endure such a custom? And did Apollonius not reform it when he was occupying

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[1] vi. 19 (3): *φαντασία ταῦτ' εἰργάσατο, σοφωτέρα μιμήσεως δημιουργός.*

[2] vi. 19 (5): *ὑμεῖς δ' ἀφήρησθε τοὺς θεοὺς καὶ τὸ ὁρᾶσθαι καλῶς καὶ τὸ ὑπονοεῖσθαι.*

himself with the affairs of the Lacedæmonians? Apollonius replies that it would have been madness to contend against a religious custom such as this. The scourging is performed in accordance with an oracle directing that the altars shall be sprinkled with an offering of human blood to the Scythian Artemis. This no doubt was originally a requirement of human sacrifice; but the Spartans, by subtly interpreting it, have at once evaded the obligation of putting a human victim to death and turned a rite which they could not get rid of into an exercise in fortitude. Thespesio, however, skilfully presses the point; ending with the remark that he has been speaking not against the Lacedæmonians but against Apollonius. If we thus rigorously investigate customary rites the origin of which reaches back to a grey antiquity beyond knowledge, and cross-examine divinities as to their reasons for delighting in them, not the Eleusinian nor the Samothracian nor any other mysteries will be safe. We can always ask "Why this and not that?" and take offence at one thing or another. In these matters at any rate, if not in all, the Pythagorean silence is good. Apollonius accordingly, relinquishing further argument on behalf of the Spartans, consents to go on to another topic, and proposes that they shall discuss the nature of justice.[1]

Such a subject of discourse, Thespesio agrees, is suitable both for professional philosophers and for others. Apollonius then recalls the comment of the Indian sages on his notion that when, being in a former body, he had refused to betray his ship to pirates, he had performed an act of justice. They laughed at this use of the word, holding that justice involves something more than the absence of injustice. Rightly, answers Thespesio, for no virtue consists in a mere negation. And we must not expect to find men publicly rewarded for practising justice. In the cases of Socrates and of Aristides we rather find the opposite. No doubt it will seem absurd: but as a matter of fact Justice, being appointed by Zeus and the Destinies to prevent men from injuring one another, takes no measures to prevent herself from being injured. Imagine,

[1] vi. 20.

however, that when Aristides returned from his apportionment of tribute among the allies of Athens, the proposal had been made by two orators to confer the crown upon him for his justice; and that one had assigned as the reason his returning no richer than he went, and the other his observance of due proportion to the capacity of each allied State, and his refraining from all excessive demands: would not Aristides himself have protested against the first orator for the inadequacy of his reason, and recognised that the second was aiming at the true mark? And indeed, in maintaining due proportion, he had regard to the advantage both of Athens and of the islands; as was seen afterwards when the Athenians, by imposing heavier burdens, brought about the revolt of their tributaries and the loss of their empire. He, then, is just who both acts justly himself and so orders things that others shall not act unjustly. And from this diffusive virtue—which is better than oaths taken on sacrifices[1]—will spring both other virtues and in particular those of the judge and of the legislator, which come peculiarly within the province of justice.[2]

To this account of the just man Apollonius assents. After some further discourse, he informs Thespesio of his intention to go in quest of the sources of the Nile. In the account of so remote a journey the geography and zoology as usual become mixed with the marvellous, though they are not wholly fictitious. We are told of the Androphagi and the Pygmies, who are of Æthiopian race, and extend as far as to the Æthiopic Sea, into which no one voluntarily sails. We also hear of cataracts haunted by dæmons; and there is a curious story about the taming of a satyr in one of the villages by Apollonius.

On his return, he signified his approval of the conduct of Titus after he had taken Jerusalem, in refusing to accept a crown from neighboring nations.[3] Titus, now associated with his father in the

[1] vi. 21 (7): *δικάσει μὲν γὰρ τοιόσδε πολλῷ δικαιότερον ἢ οἱ κατὰ τῶν τομίων ὀμνύντες.*

[2] It is noteworthy that the place here assigned to justice as a positive virtue coincides with that which it occupies in Dante's description of the spirits in the heaven of Jupiter, who are those of men that bore rule on earth.

[3] vi. 29: *μὴ γὰρ αὐτὸς ταῦτ' εἰργάσθαι, θεῷ δ' ὀργὴν φήναντι ἐπιδεδωκέναι τὰς ἑαυτοῦ χεῖρας.*

government, invited him to Argos, and consulted him as to his future behavior as a ruler. Apollonius says that he will send him his companion Demetrius the Cynic as a free-spoken counsellor; and Titus, though the name of the Cynic is at first disagreeable to him, assents with a good grace.[1] He is also said to have consulted Apollonius in private on his destiny.

Apollonius, says Philostratus at this point, made many more journeys, but only to countries he already knew. He remained always like himself; and this is for the sage even more difficult than to know himself. Before proceeding to the account of his acts and sufferings under Domitian, the biographer brings together a few miscellaneous anecdotes. One of these throws interesting light on popular beliefs in the eastern provinces of the Roman Empire. The cities on the left of the Hellespont, it is recorded,[2] being once troubled with earthquakes, certain Egyptians and Chaldæans were collecting money for a sacrifice, estimated at the price of ten talents, to Earth and Poseidon, and declared that they would not perform the sacrifice till the money was paid down. Apollonius drove them away for their greed, and by due rites quieted the earth.

Since those who adopt the philosophic life are best proved by their attitude to tyrannies, the behavior of Apollonius in face of Domitian has now to be compared with that of elder philosophers when confronted with tyrannies in their time. Philostratus proceeds to make the comparison in set form; maintaining the thesis that Apollonius showed his superiority to all others, high-minded as they had undoubtedly proved themselves. It is not his purpose to depreciate the rest, but it is his duty to show the greatness of his hero.[3]

Some of the sayings of Apollonius against the Emperor having been recorded, we are told that he fell under suspicion through his correspondence with Nerva and his associates Orfitus and Rufus. When proceedings against them were begun, he addressed to the statue of Domitian the words: "Fool, how little you know of the

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[1] vi. 31. [2] vi. 41. [3] vii. 1, 2.

Fates and Necessity! He who is destined to reign after you, should you kill him, will come to life again."[1] This was brought to Domitian's ears by means of Euphrates. Foreknowing that the Emperor had decided on his arrest, Apollonius anticipated the summons by setting out with Damis for Italy. They arrive at Puteoli, and there fall in with Demetrius, who leads them to the seat of "the ancient Cicero," where they can converse privately. Demetrius tells Apollonius that he is to be accused of sacrificing a boy to get divinations for the conspirators; and that further charges against him are his dress and his manner of life and the worship that is said to be paid to him by certain people. He then tries to dissuade him from staying to brave the anger of a tyrant who will be unmoved by the most just defence, and who is undistracted by that devotion to the Muses which, when Nero was singing and playing on the lyre, gave the world some respite. Damis, who till now has been unaware of the purpose of his master in coming, seconds the argument of Demetrius. Apollonius holds this timorous counsel excusable on the part of Damis, who is an Assyrian and has lived in the neighborhood of the Medes, where tyrannies are adored; but as for Demetrius, he does not know how he will make his apology to philosophy. He himself intends to remain; and in justification he sets forth the arguments that this is the only course worthy of his character. Of despotisms he allows that that is the most dangerous kind which, like the tyranny of Domitian, proceeds under forms of law. All the more, however, is he bound to appear and answer the charge against him: to flee from a legal trial would have the appearance of self-condemnation. And whither shall he flee? It must be beyond the limits of the Roman Empire. Shall he then seek refuge with men who know him already; to whom he will have to acknowledge that he has left his friends to be destroyed by an accusation which he has not dared to face himself? Perhaps Demetrius will tell him to go among those who do not know him. But here too, as he makes impressively clear, starting from the use of the word by Euripides

[1] vii. 9 (1).

in the *Orestes*, the power called conscience (σύνεσις) will follow him, and will allow him no peace whether awake or asleep.[1] At the end of this address, Damis recovers courage, and Demetrius, far from continuing his opposition, cannot sufficiently express his admiration of Apollonius.

The *præfectus prætorio* at that time, the narrative continues, was Aelianus, who had been acquainted with Apollonius in Egypt. As a diversion in his favor before he arrives, he argues to the Emperor that the "chattering sophists," having nothing to enjoy in life, deliberately try to draw death upon themselves at the hands of those who bear the sword. Perceiving this, Nero could not be brought by Demetrius to give him the death he desired, but let him go, not as pardoning him but out of contempt.[2] On the arrival of Apollonius as a prisoner, Aelian uses his authority to submit him to examination in secret. When they are alone, he gives expression to his friendly feeling, but explains the necessity of proceeding with caution. Apollonius asks him what he is accused of. Aelian repeats the heads of accusation already mentioned; informing him that the most serious charge is precisely that which he himself knows must be false, but which the Emperor is most disposed to believe true: namely, that Apollonius slew an Arcadian boy for sacrifice to encourage Nerva in aspiring to the empire; the offering being made at night by the waning moon.[3] In answering the charge, however, let Apollonius avoid a contemptuous attitude. The interview being at an end, Aelian calls in the guards, and, with simulated anger, consigns him to custody among those who are awaiting their trial.

In prison, he is derided by a military tribune, who tells him that *he* knows what he is accused of if Apollonius does not. He is accused of being worshipped by men and thinking himself worthy

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[1] vii. 14 (8–10). This passage is of high interest philosophically, as showing how fully the ethical conception of conscience had already been brought into view. The psychological conception of consciousness (sometimes expressed by the same word) was not so completely formulated till the Neo-Platonic period, with its more definite direction to abstract thought.

[2] vii. 16.

[3] vii. 20.

of equal honors with the gods.[1] As a test, let them go outside the walls, and he will try to cut off the head of Apollonius with his sword. If he succeeds, Apollonius is innocent of the claim to divinity. If he is terror-stricken and the sword falls from his hand, that is a proof at once of the divinity of Apollonius and of his guilt.

Here the histories are given of some of the other prisoners, who are deploring their fate. The philosopher, in accordance with his professional character, calls them together and addresses to them a consolatory discourse which gives them fortitude and hope. Telling them first not to despair before their cases are decided, he proceeds in a more elevated strain. During the whole of our life, the body is the prison of the soul; and those who dwell in palaces are more under this bondage than those whom they put in bonds. Nor is a savage mode of life a protection. The Scythian tribes are no freer than we are; but are surrounded with hardships by rivers impassable save when frozen over by the cold of winter, and shrink even within the shelter of their wagon-huts. And, if it is not puerile to recur to the fables of the poets,[2] one might tell of gods who are said to have been bound in chains, both in heaven and on earth. Think finally of the many wise and blessed men who have suffered at the hands both of licentious peoples and tyrannies, and resolve not to be surpassed by them in courage.

The next day, an emissary of Domitian comes in the guise of a much-dejected prisoner, but Apollonius sees his purpose of entrapping him, and discourses to his fellow-prisoners only of his travels. On the evening of the fifth day, one from Aelian brings him the message that he is to be led before the Emperor on the morrow; renewing the advice not to be contemptuous, and describing Domitian's appearance and manner of speaking. The fact that Apollonius had come forward to undergo danger on behalf of others, Philostratus here remarks, made a favorable impression even on those who before were prejudiced against him. While he is being led under guard to the Emperor's presence, he rallies his

[1] vii. 21 (1): *τὸ γὰρ προσκυνεῖσθαί σε ὑπὸ τῶν ἀνθρώπων διαβέβληκεν ὡς ἴσων ἀξιούμενον τοῖς θεοῖς.*

[2] vii. 26 (5): *εἰ δὲ μὴ μειρακιώδης ὁ λόγος.*

Assyrian disciple on the mortal terror he is in. Damis—who ingenuously confesses how terrified he is—is not admitted; and Domitian insists that the philosopher shall defend himself alone from the charges, and not Nerva, Rufus, and Orfitus, who are already condemned. Apollonius, nevertheless, declares them innocent, and protests against the injustice of assuming their guilt before their trial. Domitian, now telling him that as regards his defence he may take what course he likes, has his beard and his hair shorn, and puts him in fetters such as are reserved for the worst criminals. A letter attributed to Apollonius in which he supplicatingly entreats the Emperor to release him from his bonds, Philostratus pronounces to be spurious.[1]

When Apollonius has been lodged in his new dungeon for two days, a Syracusan who is "the eye and tongue of Domitian," visits him under the pretence that he is a well-wisher and has gained access to him by payment. After much feigned commiseration he reveals his drift; hinting that Apollonius can easily obtain his release by giving information about the supposed conspiracy against the Emperor. The Syracusan having gone away without result, Apollonius tells Damis that he was once that Pytho of Byzantium who came from Philip on a mission to the Greeks, and whom Demosthenes withstood at Athens. He also predicts that they will suffer nothing more than they have suffered already; and, to show that his submitting to bondage is voluntary, frees his leg from the fetter and then replaces it.[2]

These things, says Philostratus, the more foolish sort ascribe to magic; against the efficacy of which he again takes up the argument. Successful events attributed to charms or sacrifices may be more rationally explained by chance coincidence. Nothing, however, will persuade those who have recourse to such arts that success does not result from performance of the prescribed rites, while failure is to be attributed to the omission of some detail the importance of which was overlooked. Others, he adds, have ridi-

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[1] vii. 35. This letter is not among the extant epistles.

[2] vii. 38.

culed the art at large; but if the young will follow his advice, they will have nothing to do with things of the kind, even in sport.[1] As is evident, he would willingly have ascribed the superhuman powers he conceived Apollonius to have possessed to some deeper knowledge of natural causation. Imperfect as the science of the time was, and credulous as opinion was becoming, philosophic culture repudiated in theory the anti-natural conception of miracle.

Apollonius is at last set free from his bonds, and conducted back to his former prison. His fellow-prisoners welcome him on his return, and he devotes himself unceasingly to giving them counsel. Damis he now sends to Dicæarchia (Puteoli) to expect with Demetrius his appearance after he has made his defence.

When the philosopher is brought to the imperial judgment-seat to be tried, Domitian is to be figured as vexed with the laws because they invented courts of justice.[2] The court was decked out as for a festival oration, and all the illustrious were present. Apollonius, on entering, so disregarded the monarch as not even to glance at him. The accuser therefore crying out to him to "look towards the god of all men," he raised his eyes to the ceiling: thus indicating, says the biographer, that he was looking to Zeus; and thinking him who was impiously flattered worse than the flatterer.[3]

He had prepared an oration in case this should be necessary; but Domitian merely put to him four brief interrogatories. Those he triumphantly answers, and the Emperor acquits him amid applause; telling him, however, to remain so that he may converse with him in private. Apollonius thanks him; but adds a stern reproof. "Through the wretches who surround you," he exclaims, "cities are destroyed, the islands are filled with exiles, the continent with groans, the armies with cowardice, and the senate with suspicion." Then he suddenly disappears from among them.[4]

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[1] vii. 39 (3): *ἐμοὶ δ' ἀποπεφάνθω μηδ' ἐκείνοις ὁμιλεῖν τοὺς νέους, ἵνα μηδὲ παίζειν τὰ τοιαῦτ' ἐθίζοιντο.*

[2] viii. 1: *ἀνατυποῦσθαι δὲ χρὴ οἷον ἀχθόμενον τοῖς νόμοις, ἐπειδὴ εὗρον δικαστήρια.*

[3] viii. 4: *ἐνδεικνύμενος μὲν τὸ ἐς τὸν Δία ὁρᾶν, τὸν δ' ἀσεβῶς κολακευθέντα κακίῳ τοῦ κολακεύσαντος ἡγούμενος.*

[4] viii. 5 (6).

Since Apollonius composed a speech which he was not allowed to deliver, Philostratus thinks that this too ought to be set before the reader. What he gives is an elaborate defence in which the philosopher repudiates all magical arts and all claims to divinity, except so far as good men may be said to participate in the divine. His life, pure from blood-sacrifice and other pollutions, brings him nearer to the gods, and the lightness of his diet enables him to form presages and hence to be of service to men. In turning men's souls from their vices he is of use to their rulers also, who find them more governable. This being so, if the people did think him a god, the deception would be a gain to the master of the flock.[1] They did not think him a god, however, but only held the ancient opinion that by virtue men can participate in the divinity. A man who has something of divine order in his own soul can by wisdom draw away the souls of others from over-vehement desires of pleasure or wealth. For such an one, it is perhaps not impossible to withhold them from contact with murders; "but to wash them clean," adds the Pythagorean, "is possible neither for me nor for God the Maker of all."[2] He is made to refer to some of the wonders recorded in the biography; but he disclaims the possession of power to keep a dying friend in life or to recover him from the dead. Had it been in his power to do either, he would have done it.[3] In the part of his apology referring to the accusation of having said that if the Fates have determined that a certain man shall reign, then, though the Emperor kill him, he will come to life again, Apollonius points out that such assertions are of the hyperbolical kind adapted to produce conviction in those who find things that are put consistently with the appearance of reason incredible.[4] The implied view is obviously that of the Stoic determinism. If the predetermined event is infallible, its conditions are in reality equally

[1] viii. 7 (21): *ὥστ' εἰ καὶ θεὸν ἡγοῦντό με, σοὶ κέρδος ἡ ἀπάτη εἶχε· ξὺν προθυμίᾳ γάρ που ἠκροῶντό μου, δεδιότες πράττειν, ἃ μὴ δοκεῖ θεῷ.*

[2] viii. 7 (26): *φόνων γὰρ ἀνασχεῖν μὲν αὐτὰς μὴ προσάπτεσθαι οὐκ ἀδύνατον ἴσως ἀνδρὶ τοιούτῳ, ἀπονῖψαι δ' οὐτ' ἐμοὶ δυνατὸν οὔτε τῷ πάντων δημιουργῷ θεῷ.*

[3] viii. 7 (46).

[4] viii. 7 (53): *τὰς γὰρ ὑπερβολὰς τῶν λόγων ἐσαγόμεθα διὰ τοὺς τοῖς πιθανοῖς ἀπειθοῦντας.*

necessitated. Unconditional fate is an abstraction; though it is an impressive and a moralising abstraction. In conclusion, he quotes the lines of Sophocles in the *Œdipus Coloneus* on the revolutions of human life—

μόνοις οὐ γίγνεται
θεοῖσι γῆρας οὐδὲ κατθανεῖν ποτε,
τὰ δ' ἄλλα συγχεῖ πάνθ' ὁ παγκρατὴς χρόνος.

Let the Emperor remember how ephemeral is good fortune, and put an end to the oppressions through which he has been made hateful to all, as all things have been made hateful to him.

When Apollonius, as has been related, strangely disappeared,[1] the tyrant did not break out into a rage, as most expected, but rather gave signs of trouble. This having taken place at Rome before noon, Apollonius appeared in the afternoon of the same day at Puteoli to Damis and Demetrius, as he had promised. He came to them when they were beginning to despair of ever seeing him again; and convinced them by having a tangible body that he had not returned from the shades. After he has slept, he tells them that he is about to sail for Greece. Demetrius is afraid that he will not be sufficiently hidden there: to which he replies that, if all the earth belongs to the tyrant, they that die in the open day have a better part than they that live in concealment.[2] To those in Greece who asked him how he had escaped, he merely said that his defence had been successful. Hence when many coming from Italy related what had really happened, he was almost worshipped; being regarded as divine especially because he had in no way boasted of the marvellous mode of his escape.

Of this residence in Greece one singular adventure is related. Apollonius desired to visit the cave of Trophonius at Lebadea in Bœotia. The priests refused to admit him; making excuses to him personally but alleging to the people as their ground his being a sorcerer. He went, however, in the evening with his companions and forced his way in. In this he did what was so pleasing to the god that Trophonius appeared to the priests and rebuked them. To the inquiry of Apollonius, what philosophy he regarded as the

[1] viii. 8: *δαιμόνιόν τε κοὐ ῥᾴδιον εἰπεῖν τρόπον.* [2] viii. 14.

best and purest, he replied by allowing him to carry off a book containing the Pythagorean precepts. This book, says Philostratus, is now at Antium; and his own opinion is that it was brought with some of the epistles of Apollonius to the Emperor Hadrian, and left in the palace there.[1]

A concourse of disciples from Ionia joined with those of Hellas to surround the philosopher; and rhetoric lay neglected as an art that can teach only language. He kept his disciples away from the forensic orators (τοὺς ἀγοραίους); having always been hostile to them, and now, since he had seen the Roman prisons, regarding them and their money-making art as more responsible for the state of things there than the tyrant himself.[2]

About this time a crown (στέφανος) was seen around the sun obscuring its rays. The portent was fulfilled when Stephanus plotted the death of Domitian, then fresh from the murder of Flavius Clemens. Stephanus, says Philostratus, being the freedman of his wife—who was, like Clemens himself, a relation of Domitian, though not his sister, as Philostratus has it—avenged his death by attacking the tyrant with a spirit equal to that of the most freeborn Athenians. He proceeds to give an account of the tyrannicide, which, as we see, he approves in entire consistency with classical ethics. While this was taking place in Rome, Apollonius—having returned to Ionia after a stay of two years in Greece—was speaking at Ephesus. Interrupting his discourse, which had gradually become troubled, he stepped forward three or four paces and cried out, "Strike the tyrant, strike!" Then he told his audience that Domitian had been slain at that hour; and this vision of his from the gods was afterwards confirmed circumstantially.[3]

Near the end of Nerva's brief reign (96–98) he disappeared from among men, in some way that is not precisely known; for he sent Damis away when the expected time approached, on the pre-

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[1] viii. 19, 20.

[2] viii. 22. Cf. Tac., *Dial. de Oratoribus*, 12: "nam lucrosae huius et sanguinantis eloquentiae usus recens et malis moribus natus, atque, ut tu dicebas, Aper, in locum teli repertus."

[3] viii. 26.

text of entrusting him with a confidential letter to the Emperor. Damis does not even tell his age, which some make to have been eighty, some over ninety, and others more than a hundred. According to Philostratus, his statues in the temple at Tyana showed him to have possessed in a pre-eminent degree the charm which is sometimes found to accompany old age. Several legends are related of the manner in which he was called from earth.[1] He always taught the immortality of the soul, but did not encourage the indulgence of curiosity about its future. To a disputatious youth who, even after his departure, continued to argue against immortality, he appeared in a vision and delivered an oracle. If the verses[2] are by Apollonius, he would seem to have anticipated the attitude of Kant at the conclusion of his *Träume eines Geistersehers*. Philostratus lastly tells us that he has found no tomb or cenotaph of Apollonius anywhere, but that everywhere he has met with marvellous stories.

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The effect of the work of Philostratus on cultivated opinion was decisive. Apollonius was henceforth recognised as at least a philosopher and perhaps something more. Not that the marvels related produced this effect. No school was led by them to call itself after the name of Apollonius, and no one appealed to his wonder-working as evidence of the truth of the doctrines attributed to him. The feeling seems to have been—and, as we shall see, an adherent of the new religion was not entirely exempt from it—that here was undoubtedly a genuine moral and religious teacher. When, however, the struggle between Christianity and the established polytheism reached its critical point, it occurred to one advocate of the old religion to select the Life of Apollonius as containing wonders better authenticated than those appealed to by the Christians. The argument of Hierocles, so far as it can be gathered from Eusebius, was this: "You proclaim Jesus a god on account of a few prodigies recorded by your evangelists. We have writers of more education than yours and with more care for truth, who re-

[1] viii. 30. [2] viii. 31 (3).

late similar wonders of Apollonius; and yet we, showing more solid judgment, do not make him a god on account of them, but only regard him as a man found pleasing to the gods." This is practically all that Eusebius tells us about the contents of the work written by Hierocles under the title *Philalethes*. Everything else in the book, he asserts, has been urged by others and has been already replied to. The parallel between Apollonius and Christ is all that is new, and this only will be taken up. What seems especially to have stung the father of ecclesiastical history is the taunt of Hierocles about the "heedlessness and lightness" (*εὐχέρεια καὶ κουφότης*) of Christian belief, to which he recurs again and again. A brief analysis of his argument will not be uninstructive.

He will waive, he tells us, such points as this,—that the coming of Christ alone was foretold by the wise men of the Hebrews under divine inspiration, and that to this day devils are cast out by the power of his name, as the writer can testify from experience.[1] Of the biographers referred to by Hierocles—namely, Maximus of Ægæ, Damis the Assyrian, and Philostratus the Athenian—it will be sufficient to consider the last. From his trustworthiness, that of the rest may be judged. Accordingly the method of Eusebius is to examine in succession the eight books of Philostratus, pointing out in each the inconsistencies and incredibilities of the narrative. I have no objection, he says, to placing Apollonius as high as any one likes among philosophers. But when his biographer, be he Damis the Assyrian, or Philostratus, or any one else, represents him, under cover of Pythagoreanism, as going beyond the bounds of philosophy, then he is really made out to be an ass in a lion's skin, a juggling quack instead of a philosopher. There are limits set to human powers which no man may transgress; though a higher being may condescend to the conditions of human nature.

Was Apollonius then a divine being? If so, let the biographer preserve consistency through the whole narrative. He is said to have been announced to his mother before his birth as an incarna-

[1] *Adversus Hieroclem*, 4: *εἰσέτι καὶ νῦν τῆς ἐνθέου δυνάμεως τὴν ἀρετὴν ἐπιδείκνυται μοχθηρούς τινας καὶ φαύλους δαίμονας ψυχαῖς ἀνθρώπων καὶ σώμασιν ἐφεδρεύοντας ἀπελαύνων διὰ μόνης τῆς ἀρρήτου προσηγορίας αὐτοῦ, ὡς αὐτῇ πείρᾳ κατειλήφαμεν.*

tion of the god Proteus, and swans are said to have sung him into the world. Whence did the writer get this? It cannot have been from a disciple who joined him long after in Nineveh.[1] In one place he is made to describe himself as knowing all languages without learning them. Yet he is said to have acquired the Attic mode of speech by discipline and attention, and not by nature, and to have been taken by his father to a rhetorician at Tarsus. Many things related of him, Eusebius allows, are credible as belonging to the history of a wise and good man. It is the attempt to ascribe to him a nature more than human that gives ground for blaming both the author and the subject of the biography.

Passing from the first to the second book, Eusebius points out inconsistencies in the account of the journey to India and the meeting with King Phraotes. He then dwells on the marvellous tales about India related in the third book. Behold, he exclaims, the incredibilities in which "Philalethes" glories; preferring Philostratus to our divine evangelists not only as a man of highest education but as careful about truth![2] Iarchas, the chief teacher among the Brahmans, is represented as sitting, in the manner of a satrap rather than of a philosopher, on a more elevated and more adorned seat than his fellows. This outward distinction by the marks of tyrannic privilege was a fitting mode of doing honor to the teacher of divine philosophy.[3] The account by Philostratus of the vegetative growth like wool that enables the philosophers to dispense with clothing made from materials furnished by animals seems to require that we should think of them as laboring at the loom,—unless we are to suppose that this substance of its own accord changes into their sacred raiment.[4] That Apollonius praised the automatic mechanisms of the sages is inconsistent with his not caring to know of them in detail or to emulate them.[5]

Not till the return of Apollonius from India does the biographer, in the fourth book, make him begin his wonderful works. Yet, had he been of a diviner nature than that of man, one would

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[1] *Adversus Hieroclem*, 8. Cf. 12. [2] *Adversus Hieroclem*, 17.
[3] *Ibid.*, 18. [4] *Adversus Hieroclem*, 23. [5] *Ibid.*, 25.

say that he ought to have begun them long before, without need of communicating with the Arabians and the Magi and the Indians. Eusebius then scornfully comments on the account of his destroying the plague of Ephesus. The story about the ghost of Achilles, he proceeds, is also full of absurdities and inconsistencies. The ghost appears at dead of night and disappears at cockcrow; circumstances which would be appropriate enough in the case of evil demons, but are out of place when related of the soul of a hero. The "heedlessness" of the writer in his accounts of the casting out of a demon from a young man, and of the chasing away of the lamia, does not need much elaboration of proof; for this, as they say, is a casting out of demons by demons.[1] The raising of a maiden from death to a second life is most incredible, and to Philostratus himself seems a marvel to be explained away.[2] Had such a wonder really been performed by Apollonius at Rome, it would hardly have escaped the attention of the Emperor and of all his subordinates, and especially of the philosopher Euphrates who at that time was there, and who would not have failed afterwards to include this among his accusations of magic.

In his interviews with Vespasian, this steersman of an Egyptian ship—for such Apollonius told the Indian sage that he had been in a former life—gives himself the airs of a god and of a king-maker. He commends Euphrates to Vespasian; and afterwards, when he is at variance with him, speaks of him to Domitian as the worst of men. How does Philostratus reconcile this with the prescience he attributes to his hero? Evidently, if the wonders related by the writer actually took place, Apollonius performed them by the aid of a demon. Had the superhuman insight he displays on some occasions been of a divine character, he would have displayed it always, and would never have needed to inquire about anything. The fact that he foreknows some things and not others is best explained by the theory of demoniac assistance.[3] As was

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[1] *Adversus Hieroclem*, 30: *δαίμονας γὰρ ἀπελαύνει ἄλλῳ ἄλλον, ἤ φασι, δαίμονι.*

[2] *Ibid.*: *ἀπιστότατον καὶ αὐτῷ δόξαν τῷ Φιλοστράτῳ παραιτητέον.*

[3] *Adversus Hieroclem*, 35: *ὅρα δὴ οὖν, ὡς ἔφην, τὴν πᾶσαν αὐτῷ παραδοξοποιίαν, ὡς διὰ δαιμονικῆς ἀπετελεῖτο ὑπουργίας.*

said above, he could drive away a demon like the lamia by a more powerful demon.

From the accusation of magic that was brought against Apollonius his biographer is anxious to defend him. The incident in the dungeon, however, by which Damis is said to have been first convinced of his superhuman powers, if true, plainly confirms the charge. The explanation here suggested by Eusebius is that an impression made on the imagination of Damis by his master's associate demon (*ὑπὸ τοῦ παρέδρου δαίμονος*) caused him to see the fetter apparently removed and then replaced.[1] Apollonius, it is here evident, did not know the future; for he prepared a long defence which, in the event, was not needed. Moreover, if we examine that defence, we shall find a sufficient justification of the charge of magic by merely comparing his own definition of a sorcerer as a professor of false wisdom (*ψευδόσοφος*) with the things recorded of him by Philostratus. In what he says to Domitian about the words he had uttered on Necessity, he evades the true charge that he had predicted his end; and is thus placed before us as a flatterer and a liar and anything rather than a philosopher. Perhaps, however, the falsehood comes from his biographers. In that case, where are the "men of highest education" of "Philalethes"? The splendor of the truth has convicted them as plainly liars and uneducated men and jugglers.[2]

Lastly, says Eusebius, arriving at the culmination, Philostratus, having thrown doubt on the place and manner of his departure from life, will have it that Apollonius went to heaven bodily, accompanied by an unexpected song of maiden voices.[3]

Selecting now, as an example of his false doctrine,[4] the utterances attributed to him on the certainty of fate, Eusebius ends with some commonplace libertarian declamation:[5] remarking finally that, should any still think fit to place Apollonius among philosophers, he does not object, if only they will clear him of the false

[1] *Adversus Hieroclem*, 39.

[2] *Adversus Hieroclem*, 43: *ψεύστας ἐναργῶς καὶ ἀπαιδεύτους καὶ γόητας τῆς ἀληθείας τὸ φέγγος διήλεγξεν.*

[3] *Ibid.*, 44. [4] *ἡ ἐν δόγμασι ψευδοδοξία τἀνδρός.* [5] *Ibid.*, 45–48.

ornaments affixed to him by the writing under examination; the real effect of such additions being to calumniate the man himself under the guise of raising him to divinity.

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The moral of the Bishop of Cæsarea's tract is, it may be hoped, too obvious for comment. We may go on now to consider briefly an interesting problem raised by the reforming activity of the philosopher or prophet of Tyana.

Eusebius does not suggest that Philostratus himself had either a hostile or a friendly intention with regard to Christianity. Yet it seems likely that, living when he did, he had some slight bias one way or the other. One passage might be adduced in support of the former view. The declaration of Apollonius, that not even the supreme Deity can wash away the stain of murder, if it were found in Julian, could safely be set down as pointed against the Christian ecclesiastical doctrine. To Philostratus, however, it probably appeared as simply a re-affirmation of the higher ethical view, at once poetic and philosophic, against the imaginations of the multitude that by prayers or ceremonies the necessary expiation to be undergone by the soul itself—perhaps in a series of lives —can be dispensed with. This idea of an inflexible moral order, not to be derived from arbitrary volitions, severe or indulgent, was an important part of the Hellenic conception of an ethically reformed religion; but, to bring it into relief, no contrast was needed except that which Plato had drawn between the philosophic thinker on religion and the popular "medicine-man." The aim of Philostratus, in spite of his introduction of marvels, was to make it quite clear that Apollonius was not this kind of person; and indeed the position about sacrifices which by universal consent was his ought to be of itself sufficient to prove that he was not.

While there is thus nothing to show hostility to Christianity on the part of Philostratus, there is some slight evidence of a not unfriendly intention. The Syrian emperors of the third century, for whom he wrote, were themselves favorably disposed to the new religion. And in representing Apollonius as accused of perpetrating a ritual murder, may he not have meant to hint at the absurd-

ity of the vulgar accusations against the Christians? This seems at least possible. That Christianity should become the exclusive religion of the State he would certainly not have desired. What he hoped for was, we may judge, a system of toleration accompanied by ethical reform of the local cults wherever such reform might be needed. Of Christianity itself he probably knew little. He was not one of those who had caught a glimpse of the theocratic aims of the Church.[1] Indeed Themistius the Peripatetic, and Ammianus Marcellinus the military historian, had scarcely appreciated those aims in the latter part of the fourth century. Even after the victory of Christianity they seem to have still cherished dreams of a mutual toleration; taking the ground natural to sensible men of the world imbued with secular culture who saw the general agreement of all the organised doctrines, philosophic or religious, on practical morals. They could hardly have imagined that what must have seemed to men of their type so moderate and obvious a solution would have to wait, not for its triumph but for a mere beginning of its effective recognition, to the time of Locke.

The *régime* of "religious liberty," desirable as it must always have seemed to statesmen who were not bigots, has not always been practicable for governments sincerely anxious to uphold freedom of opinion. The repression of the rising Christian Church in the second century was probably, in its inception, a policy similar to the legislation of modern States against the reactionary conspiracies fomented by Catholic organisation in its death-struggle; though the exact degree of knowledge of those who attempted it, and the degree of harshness in the methods used, may be forever impossible to discern through the cloud of ecclesiastical legend.

[1] In spite of its defective information on the detail of Jewish antiquities, there is evidence in the fifth book of his *Histories* that Tacitus had gained some real insight into the spirit of intolerant theocracy which, at once dislodged and liberated by the destruction of Jerusalem, was shaping for itself a new embodiment in the incipient Catholic hierarchy. See especially c. 5. On the support furnished by theocracy to monarchy, compare what he says about the Hasmonæan kings, "qui mobilitate vulgi expulsi, resumpta per arma dominatione fugas civium, urbium eversiones, fratrum coniugum parentum neces aliaque solita regibus ausi superstitionem fovebant, quia honor sacerdotii firmamentum potentiæ adsumebatur" (*Hist.* v. 8).

An attempt to show how a more clearly conceived policy of the kind, aided instead of thwarted by accident, might have been successful in throwing Christianity back on the East, has been made by M. Renouvier in his *Uchronie.* According to M. Renouvier's hypothetical reconstruction of history, the official Stoicism retains the direction of opinion; the extra-legal power of the Emperor is gradually reduced with a view to the restoration of the Republic; slavery is brought to an end by legislation under the continued Stoical influence, instead of being left, as it actually was, to be slowly extinguished in the Middle Ages through economical causes unassisted by directing ideas. The process of return from the type of society initiated by the Cæsarean revolution being thus accelerated, Europe about the ninth century is a little in advance of what actually became its condition in the nineteenth. The empire of the West has in the meantime been resolved into a system of national republics in friendly alliance. The Christian propaganda is re-admitted when the force of the Catholic idea has spent itself in the East in mutual massacre and abortive crusading. Thus, in the hypothetical reconstruction, formal toleration of all sects, religious or philosophic, becomes at length the official system, as it is in the actual modern world after a far more wasteful struggle.

It is tempting to take this sketch as a basis and to make modifications in it by giving a more definite part than M. Renouvier does to the Neo-Pythagorean and Neo-Platonic movements. To bring about, for example, the abolition of the customs of animal sacrifice and of divination by inspection of victims, the ideas of a reformer like Apollonius were necessary; Stoicism having somewhat derogated from its philosophical character by defending the official religion as a whole. Again, to an idealist the Neo-Platonic metaphysics ought to seem an advance on the Stoic materialism. And indeed it seems clear that, in the absence of Christianity, Neo-Platonism and not Stoicism would finally have assumed the direction of opinion in the Empire. Had this been the course of events, Græco-Roman civilisation would have preserved its organic continuity, and the barbarian attack would doubtless have been thrown off. In the latter part of the second century the conservative

patriotism of Celsus foresaw that, as things were, the latent civil war kept up by the *imperium in imperio* of the Church would be fatal; that, unless the Christians could be persuaded to yield the required allegiance to the State, the whole fabric would sooner or later go down under the shock of invasion. He did not indeed foresee the recovery; but expressed the apprehension that the religion of the Christians itself, as well as true philosophy, would be submerged in universal chaos. This, as we know, did not in the end come about; though the prospect might seem near being realised in the dark centuries of the West between the end of antiquity and the beginning of new life in the Middle Age. What then would have been the result if the break-up had been averted? Would Western civilisation have assumed a fixed form analogous to those of the East though superior,—combining, let us say, the political order of China with the higher speculative thought of India and with a legal system that recognises rights as well as duties, but never developing new forms of freedom or new lines of art and thought? Or would there have been such accelerated progress as M. Renouvier has imagined?

A progressive movement might be conceived as starting from interaction between the Roman Empire and the free but undisciplined tribes of the North, when these, kept at length within their own boundaries, settled down to a life of comparative peace and began to draw their higher culture, as they would have done, from the old civilisation of Europe. We might then suppose an ethnic republic arising in the North—say, in Scandinavia—and, by offering to the South a new type for imitation instead of the city-republic of the past, leading to a system of independent national States. As the imperial absolutism, according to the hypothesis, remains unconsecrated by a new hierocracy, we should naturally suppose a transition from the monarchical to the republican form less violent than the French Revolution. Thus we should come round to M. Renouvier's result in a different manner. It would be easy to fill in details and, by selecting factors with a view to the required product, to show how every distinctive element in modern civilisation might have been evolved.

M. Renouvier himself, however, at the conclusion of his "apocryphal sketch," has sufficiently indicated at once the possibilities and the limitations of this kind of reconstruction; and the scientific interest of any such attempt cannot, of course, be in its positive result—since the result is necessarily unverifiable—though it may suggest new ways of looking at the actual process of history. We are led to see that in the complexity of real circumstances factors intervene which from time to time make continuous progress impossible.[1] Perhaps it is irrational even to desire that there should have been continuous progress; as Heraclitus thought Homer irrational for giving utterance to the aspiration "that strife might be destroyed from among gods and men"; since this would mean the destruction of the cosmic harmony itself. It is still possible to apply the teleological idea in Kant's sense to the historical process. That is to say, we may use it as a "regulative idea" to interpret history as it was; though we may not use it to inform us as to what history in general must have been. Taking it in the first sense, and using the terms of post-Kantian metaphysics, we might regard the pseudo-synthesis of Athanasius and Augustine and the rest, itself entirely without human value, as the obstacle posited by the world-soul in order to rise more explicitly to the idea of spiritual freedom. This is not of course to deny that there are gleams of borrowed light in their Kingdom of Darkness; but it is to deny the too anthropomorphic teleology of Comte, with its insistence that the Catholic ideal, as one expression of the "human providence," must have been a progressive phase in the history of humanity.[2] The immanent reason in things, being cosmic and not simply human, works in the affairs of man also through pauses like night and winter.

Such seasons, we know, bear the germs of the future; and the

[1] A recent example of this kind is the overgrowth of industrialism throughout the civilised world. It is remarkable that two poets so unlike in many respects as Wordsworth and Shelley foresaw the imminent evil of plutocracy in the early years of the nineteenth century.

[2] Comte predicted results almost purely beneficent from modern industrialism; though it must be allowed that his disciples have no more love for the present hypertrophy of commerce than other philosophers.

future is more than simply a return to a vanished past. To historical Christianity may be assigned on one side the merit of partially appropriating the idealistic metaphysic which was the legacy of Hellenic thought; and, on the other side, of preserving, in the documents to which it appealed for its authoritative dogma, elements of ethical culture which, when cleared of their dogmatic superstructure, could be seen to contain something emotionally unique. In the Hebrew prophets there is a more ardent, though not a purer and certainly not a nobler, morality than that of classical antiquity even in its final stage; and the teaching of the Gospel has become, when disassociated from a creed which was always extraneous to it, the inspiration of a more impassioned, though not of a wider, philanthropy. The first modern to bring out clearly the permanent ethical value of the Christian as well as of the Hebrew documents was Spinoza, who was enabled to do it by having discarded more systematically than any one before him the whole framework of rabbinical and ecclesiastical dogma. Since, however, the problem of making a new synthesis of the elements of ethical and intellectual culture still remains, there seems to be some advantage in returning for inspiration to more than one source. The movement of moral and religious reform from within the Hellenic world failed, owing to the circumstances of the time, as much through its merits as through any shortcomings that may be ascribed to it. Its philosophical idea of divine justice, as we have seen, was opposed to the doctrine of vicarious punishment distilled by Christian theology from the lower paganism. And for a time the original Christian teaching, such as Biblical critics now suppose it to have been, failed more tragically than the Hellenic movement. Græco-Roman civilisation indeed was broken up; and the Christian Church conquered: but, on the other hand, the genuine Hellenism has been easier to rediscover than the teaching of Jesus, which, in its association with the ecclesiastical system, became distorted almost if not quite beyond recognition. In the endeavor after restoration, may not the "Hebraist" and the "Hellenist," in the true sense of both terms, regard themselves as co-operating to a common result?

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SKETCH OF THE ORIGIN AND DEVELOPMENT OF GEOMETRY PRIOR TO 1850.[1]

[CONTINUED.]

THE intellectual movement which went out from Italy in the period of the Renaissance continued, as already said, under the direction first of France and then of England and Germany. But toward the end of the eighteenth century, after Euler had ceased to "calculate and to live"[2] and Lagrange (1736–1813) had pitched his tent in France, this country put itself once more at the head of the mathematical world. Not only by Clairaut, D'Alembert[3] (1716–1783), Condorcet (1743–1794),[4] Laplace (1749–1827), Legendre (1752–1833), Ampère (1775–1836),[5] Poinsot (1777–1859), Poisson (1781–1840),[6] and other lesser men, did this country direct the study of higher analysis and mathematical physics, but it also led scientists back to the study of the geometric forms in the sense—*mutatis mutandis*—in which that was understood by the learned men of old. This was accomplished by the work of Monge,[7] Carnot[8] (1753–1823) —disciple of Monge at the school of engineering of Mézières—and Poncelet[9] (1788–1867), the splendid triad on whose memorable pro-

[1] Translated from the Italian of Loria by Dr. George Bruce Halsted.

[2] Expression used by Condorcet in his *Éloge de M. Euler*.

[3] J. Bertrand, *D'Alembert* (Paris, 1889).

[4] Arago, *op. cit.*, 2, 1854. [5] Arago, *loc. cit.* [6] Arago, *loc. cit.*

[7] See Charles Dupin, *Essai historique sur les services et les travaux scientifiques de Gaspard Monge* (Paris, 1819); Arago, *loc. cit.*; K. Fink, "Monge" (*Korresp.-Bl. f. d. Gel. und Realschule*, Tübingen, 1892, pp. 263–289, 339–359).

[8] Arago, *op. cit.*, 1, 1854; K. Fink, *Lazare Nicolas Marguerite Carnot. Sein Leben und seine Werke nach den Quellen dargestellt* (Tübingen, 1894).

[9] See E. Holst, *On Poncelet's Betyding for Geometrien* (Christiania, 1878). Bertrand, *Éloges académiques* (Paris, 1890).

ductions we must pause an instant. The first, by uniting in one body of doctrine the few rules of perspective which the architects and artists had created for themselves, impelled by needs which they felt while cultivating their arts, and filling up with felicitous genius the many serious gaps which their ensemble presented, united his own name with a new branch of geometry, *descriptive geometry*.[1] Here is not the place to summarise the first known treatise of such branch of mathematics; we only note that if its publication had a practical and almost national motive,[2] Monge also attributed to the new science a theoretical value derived from the facility with which it enables one to conceive and then to study geometrical figures; moreover he was careful to compare his processes with those proper to analysis, showing their essential identity. By the classical works just cited and still more by the incomparable lectures at the polytechnic school of Paris,[3] of which he was one of the first and most splendid ornaments,[4] he put anew in honor the

[1] Whoever desires further particulars relating to the prehistory and history of descriptive geometry is referred to the first section of Vol. I. (Leipzig, 1884) of the *Lehrbuch der darstellenden Geometrie* of Chr. Wiener (1826–1896). Here let it suffice to mention *La théorie et la practique de la coupe des pierres et des bois ou Traité de stéréotomie* (1738–1739) of Frézier (1682–1773).

[2] In fact the *Programme* prefixed to it commences with the words: "Pour tirer la nation française de la dépendance où elle a été jusqu'à présent de l'industrie étrangère etc." (Monge, *Géometrie descriptive*, Paris, an VII.)

[3] See Arago, *op. cit.*, 3, 1855, p. 70 and following; Jacobi, "Ueber die Pariser polytechnische Schule" (*Ges. Werke*, 7, Berlin, 1891, pp. 355–370).

[4] Monge had for collaborators in his work of reform several colleagues—among others Lacroix (1765–1843) and Hachette (1769–1834)—and many scholars. Among these it is but just to make here special mention of Charles Dupin (1764–1873) who "above the others like an eagle soars," thanks specially to his two works *Développements de géométrie* (Paris, 1813) and *Applications de géométrie et de mécanique* (Paris, 1822) of which we shall have occasion to speak further on; meanwhile we here mention the eulogy written on him by Bertrand (*Éloges academiques*, Paris, 1890) and the article "Dupin" by K. Fink (1851–1898) (*Korresp.-Bl. f. d. Gel. und Realsch.*, 1893, pp. 1–27). Nor can we omit making honorable mention also of the *Mémoire sur les lignes du second ordre* (Paris, 1817) and of the *Application de la théorie des transversales* (Paris, 1818) of Brianchon (1783–1864), writings of which the first contains the fundamentals of a synthetic theory of conic sections, with special regard to their construction, founded in part on the theorem of Pascal and its correlative (which Brianchon himself made known in the XIII cah. of the *Journ. Éc. Pol.* and afterwards generalised in the Vol. IV. of the *Ann. de Math.*), in part on anharmonic ratio; while the second aims to show of what

study of geometry founded on the direct consideration of the figures which are its object, and facilitating the conception of geometrical figures of three dimensions, he prepared for the application of stereometric considerations to the research of the properties of plane figures, which Pappus had only vaguely seen (see § 5) and which represents to-day one of the most fertile methods of investigation and demonstration that geometry boasts of.[1]

By the side of the *Géométrie descriptive* of Monge must be placed the *Géométrie de position* (Paris, 1803) of Carnot, the latter having as object in common with the former to give geometry that generality of concepts and methods and in consequence that clear and disinvolved procedure which were believed to belong exclusively to algebra. The reader who knows only the title of the most important geometric work of the "organiser of victory" might believe that its subject coincides with that unfolded by the modern works on projective geometry. Nothing would be more inexact; the intention which Carnot set himself is to determine the mutual dependence which exists between the different aspects which may be assumed by a figure satisfying certain conditions in correspondence with the various positions of which the data are susceptible, a study enabling us to dispense with that minute enumeration of

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fecundity for constructions are the theorems of Menelaus and Ceva, an aim analogous to what Servois (1767–1847) (comp. J. Boyer, *Le mathématicien Franc-Courtois François Joseph Servois*, Besançon, 1895) contemplated in composing his *Solutions peu connues de différents problèmes de géométrie pratique* (Paris, 1805).

[1] The influence of Monge was not limited to France and the neighboring countries; this is proved by the writings of Garbinski (1796–1848) and Sapalski (1791–1838), viz., the *Synthetic Exposition of the Properties of Ruled Surfaces* (Warsaw, 1822) of the former and the excellent treatise on descriptive geometry (Warsaw, 1822 and 1849) of the latter. Moreover such influence was not quickly spent, since it may still be recognised in the methods of teaching elementary geometry; this is proved by the attempt to do away with the old rigorous separation between planimetry and stereometry made for the first time by Bretschneider (1808–1878) in 1844 (*Lehrgebäude der niederen Geometrie*), repeated later by Méray (*Nouveaux éléments de géométrie*, Paris, 1875), and successfully accomplished in 1884 by De Paolis in his excellent *Elementi di geometria* (Torino). With respect to the influence of Monge on the recent development of graphic methods see H. T. Eddy, *Modern Graphical Developments* (Math. Papers read at Chicago Congress, New York, 1896).

the various cases of the figure, to which the ancient geometers were constrained. Such aim is to-day attained by a path much more smooth and clear, that is by the methodical use of the signs in geometry; for this reason the *Géométrie de position* presents to-day little more than historical interest, and must be mentioned less on account of the ideas set forth, than because of the applications there given, among which we mention here the principles of polygonometry and of polyhedrometry, the theory of barycenters and the theory of transversals, a conspicuous part of which is the celebrated proposition which is to-day designated as Carnot's theorem.[1] It is well also to notice here that the memoir *De la corrélation des figures de la géométrie* (Paris, 1801) is a first sketch of the *Géométrie de position*, while the *Lettre du citoyen Carnot au citoyen Bossut* (Paris, 1799) and the *Mémoirs sur la relation qui existé entre les distances respectives de cinq points quelconques pris dans l'espace* (Paris, 1806) have with it numerous points of contact; that further the *Réflexions sur la métaphysique du calcul infinitésimal* (Paris, 1796) form an integral part of the *Géométrie de position*, as some suppose, is something that cannot be accepted without mature examination and engthy discussion.

The writings of Monge and Carnot prepared effectively for the revival of pure geometry; this must be dated from the appearance (1822) of the *Traité des propriétés projectives des figures* of Poncelet.[2] To convince the reader how memorable is this date, it is sufficient to notice that it is in Poncelet's great work that is demonstrated the power of central projection and of the principle of continuity as instruments of research and auxiliaries in demonstration;[3] that

[1] We believe it opportune to note also that Carnot uses the word *equipollence* in a sense analogous to that in which much later it was used by Bellavitis.

[2] Only in passing we mention the fierce war waged on this work when just issued, which was in a great part due to Poncelet's ideas not having been understood.

[3] Whether the ancients knew how to employ perspective in theoretical research is a question which we cannot here examine (see Chasles, *Aperçu historique*, 2d ed., p. 74, note); the analogous question concerning the principle of continuity is studied by C. Taylor in the note "On the History of Geometrical Continuity" (*Cambridge Proc.*, 1880 and 1881) and in the preface (pp. LXXIII. seq.) to the work *Ancient and Modern Geometry of Conics*, where, among others, are de-

in it the deeper study of homology in the plane and in space serves, not only as a prelude, but further as most effective preparation, for the study of univocal correspondence between two systems of equal number of dimensions (see Chapter VIII.); that there also the old notions concerning polarity[1] with respect to a conic[2] and those discoveries of Monge's school relating to a quadric surface prepare the law of reciprocity which, recognised by Vieta,[3] by Philip van Lansberg (1561–1632),[4] and by Snellius (1581–1626)[5] in the geometry of the sphere, was destined to be enunciated in all its generality and called by the name of *principle of duality*, four years later (see "Considérations philosophiques sur les éléments de la science de l'étendue," *Ann. de Math.*, 16, 1825–1826,)[6] by Gergonne, (1781–1859.[7]) While an importance rather transitory pertained to the considerations set forth by Poncelet concerning the ideal chords of conic sections,—on which one relating the history of the theory of imaginaries in geometry might pause,—on the other hand great value must be attributed to the *Supplément* of the work in question, from which, among other things, we learn of the existence of four cones in any pencil of quadrics; moreover the investigations concerning polygons inscribed in one conic and circumscribed about

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scribed the efforts of Boscovich (1711–1787) to give a firm basis to the afore-mentioned principle.

[1] The name *pole* was proposed by Servois ("Solution du premier, etc." *Ann. de Math.*, T. I., 1810–1811), that of *polar* by Gergonne ("Théorie analytique des poles, des lignes, et des surfaces du second ordre," *Ibid.*, T. III., 1812–1813).

[2] A special application of polarity is met with in the *Traité analytique des sectiones coniques* (Paris, 1707, p. 275), of the Marquis de l'Hôpital (1661–1704). See also the "Rectifications."

[3] *Variorum de rebus mathematicis responsorum liber octavus* (Tours, 1593).

[4] See K. Fink, *Kurzer Abriss der Geschichte der Elementar-Mathematik* (Tübingen, 1890), p. 196.

[5] *Doctrina triangolorum canonica* (Leyden, 1627).

[6] *Ann. de Math.*, 19, 1827, where one finds for the first time the expression *class of a curve*.

[7] Further particulars on the origin of duality are found in the *Vorlesungen über continuirliche Gruppen* by Lie, edited by G. Scheffers (Leipzig, 1893), a work in which is also set forth a most important generalisation of the dualistic relation. See moreover *Julius Plücker's gesammelte wissenschaftliche Abhandlungen*, I. (Leipzig, 1895), p. 619.

another possess permanent value; nay, if the place assigned to a mathematical research should be determined, not so much on the basis of the intrinsic value which it seems to possess, as rather by its originality and the novelty and variety of the investigations which branch out from it, it follows that to the last cited theme belongs a position second to a very few. And on this account and because it is extremely instructive to contemplate all the aspects revealed by modern mathematics in a single and minor subject, we wish to pause an instant on the so-called *polygons* of Poncelet.[1]

13. The problem: "Given in a plane two conic sections, to construct a polygon of a certain number of sides, which shall be inscribed in one of the given curves and circumscribed about the other," seems determinate, and from the year 1817 Poncelet reckoned[2] it among those to which could be applied the methods he was then elaborating; however when we remember that, if the two conics are circles the data must satisfy a certain relation[3] in order that the problem be possible, the suspicion arises that in the general case something analogous takes place. And in fact five years later Poncelet demonstrated that the problem enunciated does not admit, generally speaking, of a solution, but that when it has one, there is an infinite number (*Traité des propr. projectives*, §§ 530 et seq.; see *Applications* cited, 1, Paris, 1862, p. 348 et seq.). This memorable discovery did not pass unnoticed; which is proved by the fact that not more than six years after the time when it became public, Jacobi (1804–1851)[4] in the course of his notable researches

[1] For greater particulars refer to the "Note historique, critique et philosophique," inserted by Poncelet in the *Applications d'analyse et de géométrie, etc.* I. (Paris, 1862), to the lecture of the author, entitled *I poligoni di Poncelet* (Turin, 1889), and to the complement published in the *Bibliotheca mathematica*, 1889, with the title "Rassegna di alcuni scritti sui poligoni di Poncelet."

[2] "Réflexions sur l'usage de l'analyse algébrique en géométrie," inserted first in the *Ann. de Math.*, 8, 1817, and then in the *Applications d'analyse et de géométrie*, 2 (Paris, 1864), p. 466.

[3] In the case of a triangle this was discovered by Euler (*Nov. Comm. Petrop.*, 2, 1751) and for the other polygons it was investigated by his disciple Nicholas Fuss (1755–1826) (*Nova Acta Petrop.*, 10, 1794, pub. 1797, and 13, 1798, pub. 1827) and found in part by Steiner (*Journ. f. Math.*, 2, 1827).

[4] Lejeune-Dirichlet, "Gedächtnissrede auf C. G. J. Jacobi" (*Berl. Abh.*, 1852).

concerning the theory of elliptical functions observed—in the celebrated paper "Ueber die Anwendung der elliptischen Transcendenten auf ein bekanntes Problem der Elementar-Geometrie" (*Journ. f. Math.*, 3, 1828)—how this theory was able to furnish a complete general solution (nay, the only explicit one, that is to say, other than by a recurring way, which is known even to-day) of the problem which consists in seeking the relations existing between the radii and the distance of the centers of two circumferences which admit a polygon of Poncelet of a given number n of sides. And since, as Laplace thought, "discoveries consist in bringing together ideas capable of uniting and which were before separate," we must without hesitation give to Jacobi's observation the name of discovery. Nor need we wonder therefore at perceiving how the writing just cited was judged by Legendre worthy to be abridged in one of the supplements to the third edition of his voluminous *Théorie des fonctions elliptiques et des intégrales eulériennes*, and that Richelot (1808–1875) thought proper to devote two memoirs[1] to develop and complete the ideas of his own master and also to show how these could be applied to solve the analogous problems of spherical geometry.

At the end of the above mentioned memoir Jacobi noted that through the theory of elliptic functions it would have been most interesting to institute for two conics considerations analogous to those explained by him for two circumferences, even declaring himself disposed to return himself to the subject; but later, probably attracted by more important problems, he did not execute his project, which for nearly forty years waited in vain for some one to execute it; finally in 1865 Rosanes and Pasch gave in the notable memoir "'Ueber das einem Kegelschnitte umbeschriebene und einem anderen einbeschriebene Polygon" (*Journ. f. Math.*, 64) a transcendental solution of the problem, which consists in the in-

[1] "Anwendung der elliptischen Transcendenten auf die sphärischen Polygone, welche zugleich einem kleinen Kreise der Kugel eingeschrieben und einem andern umgeschrieben sind." (*Journ. f. Math.*, 5, 1830); "Ueber die Anwendung einiger Formeln aus der Theorie der elliptischen Functionen auf ein bekanntes Problem der Geometrie" (*Ib.*, 38, 1849).

vestigation of the relation which must be satisfied by the coefficients of the two conics in order that there may be a polygon of n sides inscribed in one and circumscribed about the other; generalising as needful the method invented by Jacobi and employing the elliptical functions of Jacobi.[1] That we can reach the same result using the analogous functions preferred by Weirstrass, was shown first by M. Simon[2] and afterwards by Halphen (1844–1889)[3]—in the second volume of his classic *Traité des fonctions elliptiques et de leurs applications* (Paris, 1888)—and by Vivanti.[4]

Having preferred in our exposition the logical order to the chronological, it is necessary to retrace our steps some decades to take note of the extremely elegant works of Nicola Trudi (1811–1884)[5] and those of A. Cayley (1821–1895)[6] not less interesting but much better known and even more widely diffused on account of being included in the collected papers of this celebrated scientist

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[1] The same analytical auxiliaries are employed in Rogers "Note on the Porism of the Inscribed and Circumscribed Polygon" (*Proc. L. M. S.*, 16, 1884–85).

[2] "De relationibus inter constantes duarum linearum secundi ordinis ut sit polygonum alteri inscriptum circumscriptum alteri" (*Diss.*, Berlin, 1867), and "Ganzzahlige Multiplication der elliptischen Functionen in Verbindung mit dem Schliessungsproblem" (*Journal für Math.*, 81, 1876). See Gundelfinger, "Ueber das Schliessungsproblem bei zwei Kegelschnitten" (*ibid.*, 83, 1877).

[3] C. Jordan, "Georges Halphen" (*Journal de Math.*, IV., 5, 1889); "Liste des travaux mathématiques de Georges-Henri Halphen" (*Palermo Rend.*, 3, 1889); Brioschi, "Notizie sulla vita e sulle opere di G. E. Halphen" (*Lincei Rend.*, IV., 5, 1889).

[4] "Sull' applicazione della funzione ellitica pu alla teoria dei poligoni di Poncelet" (*Palermo Rend.*, 7, 1893).

[5] *Sui poligoni iscritti e circoscritti alle curve coniche con date condizioni*, Naples, 1841; "Rappresentazione geometrica immediata dell' equazione fondamentale della teoria delle funzioni ellittiche con diverse applicazioni," *Naples Rend.*, 1843; "Studi intorno ad una singolare eliminazione con applicazione alla ricerca della relazione tra gli elementi di due coniche, l'una inscritta e l'altra circoscritta ad un poligono, ed ai corrispondenti teoremi di Poncelet" (*Napoli Atti*, 1863), and "Sui teoremi di Poncelet relativi ai poligoni iscritti e circoscritti alle coniche" (*Giorn. di Mat.*, 1, 1863).

[6] For biographical data concerning this geometer we refer the reader to the "Biographical Notice" by A. R. Forsyth which introduces Vol. VIII. (1895) of *The Collected Mathematical Papers of Arthur Cayley*; further to the "Notizia sulla vita e sulle opere di Arturo Cayley" by Brioschi (*Lincei Rend.*, V., 4, 1895), translated into French in *Bull. Sc. Math.*, II., 19, 1895; and lastly to the article "Arthur Cayley" by M. Nöther (*Math. Ann.*, 46, 1895).

(see Vols. 2, 3. 4, 5, 8, and 9); regarding these works we shall limit ourselves to note that from those of the English geometer we learn to write under the form of a determinant the simultaneous invariant of two ternary quadratic forms, whose vanishing announces that the conics represented by these admit of a polygon of Poncelet of the given order.

Analogous results were obtained by Mention in the "Essai sur la problème de Fuss" (*Petersb. Bull.*, 1, 1860) utilising an observation set forth by Tchébycheff (1821–1894)[1] in an article "Sur la série du problème de Fuss" (*Id., ib.*); by Puiseux (1820–1883) in the "Note sur les polygones qui sont à la fois inscrits dans un cercle et circonscrits à un autre cercle" (*Annales de la Soc. scient. de Bruxelles*, 3e ann., 1878–1879, 2e partie); by Kluyver in the memoir "Over de invariante betrekking tusschen twee kegelsneden in en om denzelfden velhoek beschreven" (*Nieuw Archief voor Wiskunde*, 15, 1888) and much earlier by Moutard in an article[2] which the originality of its concepts and the amplitude of its views places in a most eminent position among those referring to the theory which occupies us.

Lack of space forbids dwelling on the very numerous works whose modest aim was to demonstrate by methods different from those of Poncelet and Jacobi the results obtained by these, and even on the more significant ones which have for aim the study of the geometrical properties of the polygons of Poncelet.[3] But we wish to note how the theorem established by the great French geometer makes a part to-day of the splendid collection of *theorems of closure* (*Schliessungsprobleme*), to the constitution of which Steiner greatly contributed by discovering in 1832 one relative to circles on the plane or on the sphere (see *Systematische Entwicke-*

[1] A. Vassillief, *P. L. Tchébycheff et son œuvre scientifique* (Loria Bollettino, 1, 1898).

[2] "Recherches analytiques sur les polygones simultanément inscrits et circonscrits à deux coniques," in the appendix to the first volume of the *Applications* of Poncelet already cited.

[3] See in particular: Halphen, "Application de la théorie des caractéristiques pour les coniques à une question relative aux polygones de Poncelet" (*Bull. Soc. Phil.*, VII., 3, 1876).

lung etc.[1]) and one still more noteworthy relative to polygons of an even number of sides inscribed in a plane cubic (*Journ. f. Math.*, 32, 1846), to demonstrate the latter of which many celebrated geometers exerted themselves, among which it is sufficient to mention Clebsch (*Journ. f. Math.*, 63, 1864), Ed. Weyr (*ib.*, 71 and 73, 1870–1871), Schoute (*ib.*, 95, 1883), Hurwitz (*Math. Ann.*, 19, 1882), Küpper (*ib.*, 24, 1884), Disteli (*Die Steiner'schen Schliessungsprobleme nach darstellend-geometrischer Methode*, Leipzig, 1889; see also Fiedler, *Die darstellende Geometrie*, etc., III. Th., p. 352 et seq., Leipzig, 1888), Martinetti (*Palermo Rend.*, 5, 1891), Czuber (*Journ. f. Math.*, 114, 1892). Some analogous propositions are due to August (*Archiv der Math.*, 59, 1876), to Harnack (1851–1888)[2] (*Math. Ann.*, 12, 1877), to Westphal (*Math. Ann.*, 13, 1878), to Forsyth (*Proc. L. M. S.*, 14), to Juel (*Nyt Tidss. f. Math.*, 1, 1890), to J. Thomae (*Leipziger Ber.*, 47, 1895), to R. A. Roberts (*Quart. Journ.*, 29, 1898), and to others whom for brevity we omit.

Instead we observe how the examination of the collection of works referring to the theorems of closure in general gives evidence of an intimate connection between these and the theory of elliptic functions. Now, just as it has been acutely noted[3] that in mathematics the appearance of any contradiction whatsoever indicates the presence of a hidden truth capable of adjusting the momentary discord, so we may say that a point of contact between two heterogeneous doctrines must find its justification in some more general truth. Thus for the theorems of closure, this superior reason consists in this, that every one of them gives rise to a homogeneous relation doubly quadratic between two series of binary variables, that is to a relation of the same form as that which subsists between two elliptic functions with the same argument; this is what Cayley demonstrated in the memoir "On the Porism of the In- and Circumscribed Polygon" and the (2, 2) "Correspondence of Points

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[1] Comp. Vahlen, "Ueber Steiner'sche Kugelketten" (*Zeitschrift*, 41, 1896).

[2] A. Voss, "Zur Erinnerung an Axel Harnack" (*Math. Ann.*, 32, 1888).

[3] H. J. Stephen Smith, "On the Present State and Prospect of Some Branches of Pure Mathematics" (*Proc. L. M. S.*, 8, 1876, p. 25).

on a Conic" (*Quart. Journ.*, 11, 1871).[1] And these observations disclosing that part of the reasoning common to all the demonstrations of the theorems of closure, led Hurwitz to write a memoir[2] in which we know not whether more to admire the breadth of view or the perfection of form, and with which we shall end this digression, for in vain we should seek a more worthy close.

14. The papers of Poncelet "Mémoire sur les centres de moyennes harmoniques" (*Journ. f. Math.*, 3, 1828) and "Mémoire sur la théorie générale des polaires réciproques" (*ibid.*, 4, 1829), as well as the later "Analyse des transversales appliquée à la recherche des propriétés projectives des lignes et des surfaces" (*ibid.*, 8, 1832), make us approach the year 1837, in which was published the *Aperçu historique sur l'origine et le développement des méthodes en géométrie* by Michel Chasles (1793–1880)[3], a fascinating work in which the author, after having set forth in a style the beauty of which may be equalled but not excelled, all that constituted in his time the patrimony of pure geometry, vigorously defended the rights which this had to the consideration of scientists and which were continually questioned by the blind worshippers of analysis. One must not suppose however that this is a work of polemic alone, and that therefore it has, to-day that the strife is over, only a historical value. In fact from the *Mémoire de géométrie sur deux principes généraux de la science*, that follows the historico-critical part of the work in question, we learn the general properties of the collinear and reciprocal transformations whether in general or in the cases in which they are involutory, while in the thirty-four notes which accompany it are given important historical and scientific investigations. Of the first it is not necessary here to make explicit mention, and as to the others it will suffice to limit ourselves to

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[1] See also Chapters IX–X of the second part of the above-mentioned *Traité* of Halphen.

[2] "Ueber unendlich-vieldeutige geometrische Aufgaben, insbesondere über die Schliessungsprobleme" (*Math. Ann.*, 15, 1879); see also the note of the same geometer "Ueber die Anwendung der elliptischen Functionen auf ein Problem der Geometrie" (*ibid.*, 19, 1881).

[3] See J. Bertrand, "Éloge de Michel Chasles" in *Revue scientifique* of March 24th, 1892.

name those on cross ratio, on involution and on the law of continuity, since of the remainder, being more special, it is better to postpone the examination to a more opportune moment.

But in the interval of time interposed between the appearance of the writings of Poncelet and the publication of the work of Chasles, Germany had awakened from the torpor in which for half a century the soporific works of the "combinatorial school"[1] had immersed her,—a torpor from which the powerful voice of Gauss[2] (1777–1855), the *principis mathematicorum*, was not able to rescue her, he whom Jacobi did not hesitate to compare with Archimedes, finding in his works discoveries equally profound with those of the ancient, set forth in a form the most perfect, and with ideal scientific rigor; he whose mental preëminence is manifested, as Borchardt notes, in the decisive influence which he exercised on the mathematics of our time, penetrating in all his researches down to the very heart, clarifying and amplifying the fundamental concepts of mathematics, binding together under general laws facts which had remained unexplained or isolated, and uniting the rigor of the antique methods with the free play of modern analysis.

The reawakening in Germany of the spirit of investigation is marked by a new transference of the mathematical sceptre, which may be considered as having taken place in 1826, the year of the founding by the exertions of A. L. Crelle (1780–1855) of a periodical publication to which Abel (1802–1829)[3] and Jacobi, Steiner and Plücker, Moebius and von Staudt quickly insured renown not destined to decline; and it is precisely to these last four scientists that geometry is indebted for her return in honor to the east of the Rhine: to these it is therefore our duty to dedicate some lines of this introductory chapter.

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[1] The work done by this curious school who consider the polynomial theorem as the most important proposition of all analysis is analysed in Gerhardt, *Geschichte der Mathematik in Deutschland* (München, 1877) pp. 201–206.

[2] See Sartorius von Waltershausen, *Gauss zum Gedächtniss* (Leipzig, 1856); Schering, "Zur Feier der hundertsten Wiederkehr von Gauss' Geburtstag" (*Gött. Nachr.*, 1877).

[3] Bjerkness, *Niels-Henrik Adel. Tableau de sa vie et de son action scientifique* (Paris, 1885).

15. The most important of the works of Moebius (1790–1868)[1] in the field of geometry is that having for title *Der barycentrische Calcul* (Leipzig, 1827)[2]; herein the ancient ideas concerning the mass-center of a system of points are made the foundation of a most important algorithm, which leads to a new system of coördinates, of which the author demonstrates the applicability to the study of plane and twisted curves and to the research of the properties of new geometric transformations. Later applications of the same methods are contained in memoirs which the author inserted in Vols. 5 (1830), 24 (1842), 26 (1843), and 37 (1844) of the *Journ. f. Math.*, while new species of transformations (circular affinity, involution, symmetry, elementary transformations) were set forth by him in other writings, of which we do not cite the titles here in detail, nothing being easier than to find them now that the issue of his *Gesammelte Werke* has been completed. Others of his researches of a different kind will be mentioned later; here we make an exception only in favor of those on the signs of plane and solid figures (which among other things led to the ideas of polyhedra without volume and of unilateral surfaces), on the polyhedra, on the *Ausdehnungslehre* of H. Grassmann (1809–1877) and on the *Quaternions* of W. R. Hamilton (1805–1865), there being no point in our history more opportune for their mention.

Möbius contributed also to the progress of mechanics, of optics, of astronomy, as well as analysis, but these works are without the frame of our picture; if we mention them it is to indicate how they proclaimed a substantial difference between him and another of the stars of first magnitude which in that epoch illuminated the sky of German mathematics, J. Steiner (1796–1863)[3], who was so exclusively geometer that he was never willing to come to terms openly

[1] See the bio-bibliographic sketch prefixed by Baltzer to *August Ferdinand Möbius' gesammelte Werke* (Leipzig, 1885–1887).

[2] Note how a preceding work on "Zwei geometrische Aufgaben" (inserted in a volume of *Beobachtungen auf der K. Univ.-Sternwarte zu Leipzig*) shows that from 1823 Möbius knew and knew how to employ in a masterly manner the barycentric calculus.

[3] See C. F. Geiser, *Zur Erinnerung an Jacob Steiner* (Zurich, 1874); Graf, *Der Mathematiker Jacob Steiner von Utzendorf* (Bern, 1897).

with the analysts.[1] All know how in his *Systematische Entwickelung der Abhängigkeit geometrischer Gestalten von einander* (Berlin, 1832) "was revealed the organisation by virtue of which the most varied phenomena of space are connected among themselves," how here are rigorously established the principles by means of which that great geometer was able to accomplish the wonderful discoveries which will be separately cited in the course of our narrative; here we wish only to mention, in the first place how the possibility of founding on these a complete treatment of the conic sections appears from the *Vorlesungen über synthetische Geometrie* (Part I., 2nd ed., Leipzig, 1875; Part II., 3rd ed., *ibid.*, 1898) which two eminent disciples of Steiner (Geiser and Schröter) published under his name, and in the second place how he on several occasions demonstrated that pure reasoning, independent of calculation, can also be adapted to the measurement or the comparison of areas and volumes, and presented the most memorable example of this in the memoirs "On maxima and minima," to demonstrate the value of which it suffices to say that the calculus of variations was able to find only a long time after Steiner and by the way opened by him the means to follow synthesis in the solution of such questions.[2]

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[1] As is known, in the works of Steiner there are found innumerable propositions only enunciated; a complete catalogue of the writings devoted to their demonstration would prove extremely useful and is much to be desired.

[2] The original memoirs of Steiner on maxima and minima were completely published only after his death among his complete works; at first fragments of them had been published in the *Journ. f. Math.* and in the *Journ. de Math.* In this line of research Steiner was followed by Lindelöf ("Propriétés générales des polyhèdres qui, sous une étendue superficielle donnée, renferment le plus grand volume," *Math. Ann.*, 2, 1870, or *Petersb. Bull.*, 14, 1870; "Quelques problèmes relatifs à l'éllipse et à l'éllipsoïde," *Nouv. Ann.*, II., 10, 1871), by Berner Bauer ("Ueber Maxima and Minima geometrischer Figuren," *Zeitschr. f. Math.*, 11, 1866), by R. Sturm ("Bemerkungen und Zusätze zu Steiner's Aufsätzen über Maximum und Minimum," *Journ. f. Math.*, 96, 1884; "Würfel und reguläres Tetraeder als Maximum und Minimum," *ibid.*, 97, 1884), by Sturm and Lampe ("Ueber das Minimum des Inhaltes eines Vierecks bei gegebenen Seiten," *ibid.*), by Schwarz ("Beweis des Satzes, dass die Kugel kleinere Oberfläche besitzt als jeder andere Körper gleichen Volumens," *Götting. Nachr.*, 1884), and by E. Kötter ("Ueber diejenigen Polyeder die bei gegebener Gattung und gegebenem Volumen die kleinste Oberfläche besitzen," *Journ. f. Math.*, 110, 1892).

Less exclusive than Steiner was C. G. Von Staudt (1798–1867),[1] who did not completely separate himself from the influence of his great master Gauss and dedicated a part of his own intellectual activity to researches on the theory of numbers. A part, but not the most elevated and decisive, for this was absorbed by the solution of the great problem to treat the geometry of position without introducing any metrical concept; the *Geometrie der Lage* (Nürnberg, 1847), and the *Beiträge*, with which he adorned it in 1856, 1857, and 1860, certify to the complete result of his assiduous and genial efforts to rid of any extraneous element that projective geometry of which a little while before Poncelet and Chasles in France, Steiner and Möbius in Germany had laid the foundations. We cannot in few words delineate the content of these writings which justify for Von Staudt the enviable epithet of the "Euclid of the nineteenth century"; we shall only mention the new definition of projectivity there given, which contains only what is necessary and sufficient, and those of the conic and of the quadric, which, including also imaginary curves and surfaces, are able to rival the analytical definitions, and like these are susceptible of being extended to curves and surfaces of any order; we shall add that the researches on the imaginary in geometry[2] availed to put to flight the "spectre" which had pursued Steiner in the last years of his life, and to render superfluous the "principle of continuity" which had multiplied the opponents of the doctrine of Poncelet.[3] Other minor works of

[1] See the study by C. Segre prefixed to *Geometria di posizione di C. G. von Staudt, trad. dal tedesco a curo del Dr. M. Pieri* (Torino, 1888).

[2] Comp. A. Ramorius, "Gli Elementi imaginari nelle Geometria" (*Giorn. di Mat.*, 35, 1897, e 36, 1898).

[3] Among all the investigations of Von Staudt these are undoubtedly the most abstruse; on this account their results spread with more difficulty in the mathematical world; to facilitate their comprehension, efforts were made in various directions by Lüroth (*Math. Annalen*, 8, 1875, and 11, 1877), August (*Programm der Friedrichs-Realschule*, Berlin, 1872), Stolz (*Math. Ann.*, 4, 1871); Henry J. Stephen Smith (*Ann. di Mat.*, II., 3, 1869–1870), H. Wiener (*Rein geometrische Theorie der Darstellung binärer Formen durch Punktgruppen auf den Geraden*, Darmstadt, 1885), Segre (*Torino Mem.*, II., 38, 1886, and *Journ. f. Math.*, 100, 1886) and Servais (*Belgique Mém.*, 49, 1896). To the theory of imaginaries of Von Staudt is allied the "Rechnung mit Würfen," to which Lüroth (*Mem. cited*), Sturm (*Math. Ann.*, 9, 1876), Schröder (*ibid.*, 10, 1876) and G. Kohn (*ibid.*, 46,

Von Staudt contain applications of the aforesaid doctrines, and, demonstrating how he knew duly to appreciate and in a masterly manner to treat metrical questions, make us regret that it was not given him to give to the *Geometrie der Lage* a sister work in the *Geometrie des Masses* which he had projected.[1]

A direction completely different from the investigations of Von Staudt have the publications of Julius Plücker (1801–1868),[2] to whom analytical geometry is indebted for decisive advancements; to him in fact we owe the development of homogeneous and polyhedral[3] coördinates, of the coördinates of the straight in the plane[4] and of the plane in space;[5] to him, omitting for a moment

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1895) have devoted special researches. In this last work is extended the concept of *Würfe* to a group of n elements of a form of I species; such extension is applied in the later work of the same geometer entitled "Die homogenen Coordinaten als Wurfcoordinaten" (*Wien Ber.*, 104, 1895). A theory of imaginary elements different from that of Von Staudt is created by A. Mouchot and expounded in the works: *La réforme Cartésienne étendue aux diverses branches des mathématiques* (Paris, 1876) and *Les nouvelles bases de la géométrie supérieure* (Paris, 1892).

[1] It is by inspiring themselves with the conceptions of Von Staudt, or rather by unfolding them excellently, that Juel (in the dissertation "Bildrag til den imaginaere Linies og den imaginaere Planis Geometri," Kopenhagen, 1885, and in the article "Ueber einige Grundgebilde der projectiven Geometrie," *Acta*, 14, 1889) and Segre (in the interesting group of notes on "Un nuovo campo di ricerche geometriche," inserted in *Torino Atti*, t. 25 and 26, 1890 and 1891) discovered new correspondences and new figures which it is necessary to consider to exhaust completely the projective geometry of the plane (that is with real and complex points). In the same direction Sforza proceeded in writing the "Contributo alla geometria complessa" (*Giorn. di Mat.*, 30, 1892).

[2] See A. Clebsch, "Zum Gedächtniss an Julius Plücker" (*Götting. Abh.*, 15, 1872). Beltrami justly notes to this discourse that "the best eulogy which can be made of Plücker, considered as a geometer, is this, that Clebsch was not able to weave the account of his works, without recounting to a large extent the history of modern analytical geometry" (*Giorn. di Mat.*, 11, 1873, p. 153). Besides numerous memoirs published mostly in the *Journ. f. Math.*, we are indebted to Plücker for five great geometrical works; they are, *Analytisch-geometrische Entwickelungen* (Essen, 1828–1831), *System der analytischen Geometrie* (Berlin, 1835; see the *Anzeige* given of it by Plücker himself in *Journ. f. Math.*, 10, 1833), *Theorie der algebraischen Curven* (Bonn, 1839), *System der Geometrie des Raumes* (Düsseldorf, 1846), and *Neue Geometrie des Raumes* (Leipzig, 1868–1869).

[3] "Ueber ein neues Coordinatensystem" (*Journ. f. Math.*, 5, 1829).

[4] "Ueber eine neue Art, in der analytischen Geometrie Punkte und Curven durch Gleichungen darzustellen" (*ibid.*, 6, 1829).

[5] "Note sur une théorie générale et nouvelle des surfaces courbes" (*ibid.*, 9, 1831).

the geometry of the straight in space of which we shall treat *ex professo* later, we owe finally varied and most important applications of the "method of abridged notation"[1] of which he is one of the creators,[2] and of that of the "enumeration of constants" which, often but not always, he knew how to employ fitly.[3] If we here dispense with enumerating at this moment the new results for which our science is indebted to him, it is that we consider it much more convenient to do it in describing the successive evolution of the individual theories which constitute modern geometry, and to which it is now time for us to turn, having finished the sketch of the intellectual movement which prepared the present epoch. We thus shall see how the great men of whom we just now learned have been followed by a numerous and brilliant cohort of disciples, who, gleaning in the fields plowed by the masters, proved the fecundity of the seed which these had sowed.[4]

G. Loria.

University of Genoa.

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[1] "Ueber ein neues Princip der Geometrie und den Gebrauch unbestimmter Symbole und Coefficienten" (*ibid.*, 5, 1829); "Analytisch-geometrische Aphorismen" (*ibid.*, 10 and 11, 1831); "Ueber Curven dritter Ordnung und analytische Beweisführung" (*ibid.*, 34, 1847).

[2] Of this procedure (invented also by Bobillier) Plücker indicates the most valuable qualities by the following words: "Meine Gleichungsformen sind vollständige Darstellungen graphischer Constructionen, in denen nichts Fremdartiges sich findet; es sind ideale, mit analytischen Symbolen hingezeichnete Figuren." (*Journ. f. Math.*, 34, 1847, p 332.)

[3] No one ignores how dangerous is this artifice, otherwise very fertile (see for this the recent memoir published by Küpper in the *Math. Ann.*, 32, 1888); Plücker, who knew and boasted of its qualities, knew moreover its drawbacks and often succeeded in avoiding them in the manner described by Clebsch in the afore-mentioned Commemoration.

[4] The foregoing *Sketch* may be followed by other extracts from Loria's work, giving the history of modern geometry as above indicated. But the editors are at present unable to promise how much will be offered to their readers.—*Ed. Monist.*

RELIGION IN FRANCE.

INTRODUCTORY.

WHAT forms does the religious sentiment take in the various provinces of France and in the various classes of the French nation? How is this sentiment satisfied by existing positive religions? These are the questions which I propose to answer in the present paper. I make no pretence to settling the matter, and the inquiry of which I shall here give the results will doubtless throw but a pale light on the religious soul of France. To the set answers obtained from my *questionnaires* I shall add the thousand and one little facts of my daily observation and reading, hoping from these sources to arrive at a tolerably exact view of the situation.

If it should be asked in what manner a religion can satisfy the religious sentiment,—that particular state of the soul which involves fear, a sense of the beautiful, phantasy, and the need of knowledge, and which is apparently compounded of all forms of desire,—we shall at once discover that dogmas and beliefs are not alone sufficient. A religion acts, and acts powerfully, through the personal quality of its ministers, through the forms of its observances, through the architecture of its churches, and the character of its ceremonies.

It would be useles to insist on the importance of the monuments and the ceremonies of the Catholic religion; the same flight of faith that gave birth to the cathedrals still inspires respect for it among us. Not one of the least causes of the disfavor with which the Reformation was received in our country was the indiscreet zeal of the Calvinists in demolishing statues and mutilating the portals of the sacred edifices; the genius of the nation felt itself

wounded in its dearest living creations. The absence of all pomp in the reformed religion and the bareness of its places of worship were also a cause of estrangement; the popular soul did not find in the new faith the spectacles which charmed it, the joyfulness of its old festivals, and the wealth of art which had become a common possession even of the suffering and unfortunate multitudes. There is a body of sentiment so deeply rooted in the Latin race that even its most indifferent representative, when contemplating the magnificent monuments of the past, cannot but feel some vestige of the ancient emotions, and his eyes still cling to their solemn grandeurs even when their religious significance would never affect him.

I believe, therefore, that I am justified in saying that the same frame of mind which led our people to oppose the reformation of Calvin still continues to estrange us from it. It must also be acknowledged that Protestantism, even though it offers some satisfaction to the reason and gives wider scope to criticism, is still in many respects inferior to Catholicism. Its artistic infertility alone is sufficient to justify this statement.[1] It was never able to renew in France the miracles of the preceding centuries. Even in the countries where it triumphed, it could only bestow upon the edifices which it occupied that appearance of "devastated churches" which so forcibly impressed French, Italian, and Spanish travellers and reawakened in them, if only for a moment, the Catholic sentiment, as they stood in the presence of that "bare and sombre religion which saddens both the eyes and the heart."[2] Finally, coming as late as it did, it could make no pretension to satisfying the intellect of the cultivated classes. Human reason, already exacting in its demands, clamored for more than Protestantism offered, and strong minds could not find in the new belief the complete enfranchisement for which they longed on leaving the old.

The reformed religion came, therefore, not as an alleviation,

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[1] The only exception perhaps is music, and that principally among the followers of Luther. Luther possessed a warmth and feeling for art which Calvin utterly lacked.

[2] Emilio Castelar.

If it did not devote itself to the incessant guidance and unremitting surveillance of souls, it nevertheless placed them under severe constraint. It asked greater intellectual effort from its devotees, but it restricted this effort to its own sphere. It took from the imagination more than it granted to reason; it appeared to sacrifice the Gospel to the Bible, and the inspiration of the Greeks and Romans to that of the Jews. And for this reason it was less suited than Catholicism to the Latin nations and the Latin culture, which had remained semi-pagan, with little liking for theological discussions, more delicate and more full of life, fanatical by passion rather than by faith, and at bottom more indifferent.

It is true, the power of creation which signalised the Middle Ages seems to have been long exhausted. The decline of Christian art began with the Renaissance of ancient letters, and the decline attained its greatest extreme in that age of rationalism *par excellence*, —the eighteenth century. The favor which was later bestowed on Catholic architecture, which had been temporarily decried, and on the arts which were associated with it, was not accompanied by a like return to the mediæval faith. The same zeal was bestowed upon all the work of reconstructing the past; it was the historical passion of the century just passed,—a work of erudition and not of faith.

To-day the debasement of Christian art is only too patent. The religious imagery is wretched, the sacerdotal garb is at times ridiculous, and the liturgy is disgraced by vulgar songs. I have known ecclesiastics who preferred the mediocre structures of to-day to the venerable edifices of the twelfth and thirteenth centuries. the Church of the Trinity to Notre Dame. The genius of Catholicism is no longer fecund, and only partly sustains itself in the magnificent frame-work of its past.

Yet some signs of reformation manifest themselves. Here and there protests arise against the bad taste and the painted fetishes now in vogue; attempts are made to restore the plain chant and the Palestrinian music. A faint renascence is gaining ground in religious art, coincident with a return of zeal, of which we shall speak later on. This movement, insignificant as it is, cannot es-

cape the observation of thinking people. The battle having been lost by them at many different points, though they have not abandoned even these, Catholics appear to have discovered by instinct that their supreme power lies in the revivification of that moral inspiration which has penetrated art and life in so many ways.

I shall not undertake here to examine whether the quality of the French clergy—another factor which contributes to the success of a religion—is better or worse than it was in the preceding centuries. I refer, of course, to the mass of the secular clergy, for it is only too evident that we no longer have among us a Bossuet or a Fénelon. The simple fact that it is recruited from the lower classes of the nation alone suffices to explain why talents, not to speak of virtues, are growing less and less common among them.

To the same cause doubtless is due that restoration of fetishism and low devotion of which we have seen so many instances in the last fifty years. The education which the young abbés enjoy in common is not calculated to change their native aptitudes nor to make one who has taken orders through constraint or indifference the peer of him who has entered the priesthood from zeal or piety.

It is possible that the Protestant clergy, having been chosen from a minority, counts fewer mediocre men than the Catholic clergy,—the parochial clergy at least. Yet, I have reasons for believing that, taking it as a whole, it is not superior; and I cannot discover in the theses presented to the Protestant faculties of theology any convincing proof of a greater liberality of mind.

From careful observation of the different modes of worship current in the Catholic Church, one might be justified in saying that while Protestantism favors individualism in faith, Catholicism favors it in prayer. But individualism in faith presupposes theological discussions for which the Latin race has no taste. It also involves incertitude, which the generality of women cannot endure. They find freedom in the effusions of the heart; whereas the religious man finds it in silent meditation.

As for the unbeliever, he remains as far removed from the one Church as from the other. In some cases he is absolutely ignorant

of both, and we have with us to-day a class of young people which formerly did not exist. Reared in families who do not belong to the Church and withdrawn since childhood from all proselytic influences, these people have neither religious thoughts nor religious habits. Even the contact of ordinary life scarcely affords them a glimpse into the state of the Christian soul, of which they have no understanding. They are not extravagantly irreligious; they have neither religion nor philosophy.

This new generation is distinguished from the preceding by a very significant characteristic. It represents, so to speak, an unknown quantity in our problem. We shall attempt later to determine it.

We may now proceed to verify and extend the conclusions which we have above briefly intimated. It remains for us also to gain a closer insight into the real nature of the religious sentiment, without stopping to define how it depends or can depend upon the general state of sensibility of each individual.

I.

The Christians of France, especially the Catholics, may be divided into four distinct classes.

The first includes the routine devotees, who have no peculiar merit, and whose religion often borders on idolatry or fetishism. The second embraces a different class of believers, the devotees by habit, a more enlightened type, who accept their religion with simplicity and honesty and among whom the religious sentiment gives evidence of the higher aspirations of the human soul. To the third belong believers of the intellectual stripe, who reason out their faith or receive it from some inward light, combative or meditative, according as the trend of their mind leads them toward theology or mysticism. The fourth embraces those who doubt, who have tired of compromising with themselves and their beliefs, and are on the point of separation. With these the decomposition of faith begins and gradually continues to that final stage where a new doctrine takes the place of the ancient faith.

The first of these classes embraces the great mass of Catholics. Its only strength is in its numbers, and we shall see that it does not appreciably change in quality when it runs into unbelief. It happens also that devotees of a higher class may touch this lower stratum in some particular feature and have retained something of its intellectual condition. The old ethnic base from which it draws its sustenance is the same as that whose superstitious practices Cæsar denounced. It appears here and there in all classes of French society.

We have only to look at the crowds that flock to the places of miracles: they are composed of all classes of people. Here, at least, credulity finds its excuse in misery of body or distress of mind. Life is so full of misfortune, man suffers so cruelly both in his own person and in those whom he loves, that it is unbecoming to scoff too rancorously at these weaknesses of the heart and these vain hopes. But what shall we say of the despicability of the cult of St. Anthony of Padua, which has reached so remarkable a climax that the curate of a great parish in Paris refused to receive the image of this Saint into his chapels!

A case has come under my notice, among others, of an educated and well-born man, the son of a disciple of Voltaire, who never shot a hare without promising a coin to this same St. Anthony; he keeps open account with the treasurer of the beatific one, and absolves his indebtedness each month in cash.

And numberless are the devotions of which the poetical and tragical character alone cloaks their paltriness. Such are the vows taken in times of peril, the consecration of infants to the Virgin under the two colors, a sort of first fruits offered to Heaven. The pagan origin of these practices is too well known to be recalled here. Perhaps they are more common in the south of France, which remained more Latin, and through the Latins they may be traced back to the savage populations.

Henri Barth speaks of an African hamlet where the women, believing the camels of their caravans to be sacred beings, crawl under their bellies in order to obtain their favor. Such are the practices at the festival of St. Pancras in a village of the Provence,

where the little children are made to pass under the car carrying the image of the Saint.

I shall not stop to enumerate these practices. Catholicism has accepted and preserved them, not having been able to abolish them so soon from the hearts of the people, where they have dwelt indelibly engraved for many centuries. The people, furthermore, love the charm of them, even though their faith is ordinarily not affected by them. Their religion, reduced to such practices, is nothing more than a body of acts and gestures learned by heart and of formulas mechanically repeated.

II.

Our second class stands on a higher plane than this quasi-idolatrous mass. Nevertheless, certain traits unite the two, for we all, whether believers or unbelievers, ultimately carry within us thousands of vestiges of things long past. We are concerned here with the true Catholics, whose direct testimony appears to me the most instructive.

A gentleman of Anjou, a man of ability and character, calls attention to the delight (and this, let it be noted, expresses the sentiment of many others) which he always experiences and which has grown more intense with years, in "feeling himself borne upon this bark of Peter" which, although assailed often by furious tempests, is bound, through the guidance of its infallible navigator, finally to conduct him into port. "Doubt," he says, "is so sad and so depressing that I pity with all my heart those whose existence it darkens, and I thank God for having spared me and mine this calamity."

Another inhabitant of Anjou not so high in the intellectual scale, who has always been a believer and has grown more so with advancing age, distinguishes the religious sentiments from the human sentiments, on the ground that "the former afford a genuine, sweet, and profound peace"; while the others, on the other hand, "are always mingled with uneasiness and vague impressions, because we know that natural happiness is transitory, and cannot fill completely an insatiable heart." To preserve his faith he has

frequent recourse to the sacraments and cultivates the habit of "humble and fervent" prayer. His object is the possession of eternal life. He feels the need of zealously acting and struggling for the greatest glory of God, "whom we wish to make known and beloved all around us." The teachings of religion lead him "to practicing charity in all the sublimity of the word." He finds in this existence, "deprived of satisfactions and joys in the eyes of the world," a real happiness in this life, as he expects it in the next.

A woman of high character, belonging to the nobility of Anjou, who has passed through much suffering, likewise regards the Catholic religion as "very necessary" to her soul. "The thoughts and the emotions," she writes, "having as their objects virtue, duty, and death, lift the soul and its aspirations into a supernatural world, leaving far below it not only base and wretched thoughts, but also those that are purely human and material." She has never faltered in her religious faith. If she did not firmly believe that her religion was the only true one, she would change it. She prays and practices her religion regularly. "All the acts," she declares, "all the thoughts of a soul profoundly impregnated with the grand truths of religion, should have as their goal the eternal reward of the life to come."

A lady who lives in Paris, the mother of a family and the wife of a wealthy magistrate, shows a more combative humor. She appears to have read much, cites passages from Buisson and from Jaurès, *Les Annales de philosophie chrétienne*, a communication from Canon Pastoret to the Congress of Bourges. Very reserved in her confidences, she is still emphatic and does not analyse her thoughts. She feels religious "in concentration, silence, and meditation, or when some profound emotion,—joy, grief, affection, or admiration,—strikes her soul." "Doubt," she thinks, "produces nothing. I seek the truth; I have faith in it; I make it the basis of my life. Catholicism is the religion that comes most directly from Christ, and it is that which offers the highest ideal of perfection." She says of the Gospel: "The more I ponder it, the more it offers to me all that enlightenment to which the human soul alone can attain." And again, "God placed man in the world to *try* him (that

is, to put him to His tests). Let us be brave and resigned; let us make ourselves useful; let us aid our brethren. The difficulties of the way are slight when God guides us."

Another woman, carefully educated and very tolerant, has been living in Algiers; she has come into contact with all religions, has admired the sincere piety of the Mussulmans, and felt "how a religion is necessary to man in order to aid him to support exile." It is in her hours of suffering that she throws herself into the arms of God, who protects and consoles her. She takes delight in the pomp and ceremony of the Catholic cult, and seeks out the silence and darkness of its chapels, there to offer up her prayers. Her devotion is not painful or exact. It is an intense solace for her to possess the assurance that she will see again those whom she has lost, and that she is also able by prayer to render them some assistance.

A more simple communication, which also comes from an Angevin lady, unmarried, strikes the same note, but with a more delicate accent. She affirms the necessity of a superior being, the need of justice, the need of believing that the beings whom we have loved so much have not been brutally torn from us forever. The distinguished person who here speaks to us feels nearer to God and to religion in grief than in joy. Tribulations, were she without faith, would only find in her "resistance." A believer from infancy, she had some doubts regarding the Church and its commandments between her twentieth and twenty-fifth years; to-day she reposes in the "faith of the charcoal burner," which appears to her "delicious in its abandonment and simplicity." The religious observance that is to her most necessary is reading and the elevation of the soul, the abandonment of self to God "by thought rather than by words." She loves beautiful ceremonies and beautiful sermons; she has a horror of the "petty pieties" that belittle religion. With her religion for God is mingled a "religion of beauty"; she seek for beauty after the manner of Ruskin, passionately, in sentiment, nature, and art. Without these grand religious thoughts, she says with vigor, a woman cannot remain "mistress of herself," after she has lost inevitably the illusions "which

she loves." This is the religion which enables a woman to do her duty as the mother and educator of her children. As for her, since "henceforward there will be a vacancy in that respect" in her life, she gives it to the poor, to the unfortunate, "as much from natural pity," she adds, "as from religion."

Here is the case of two young girls, two unusual characters as I am assured, who have been successful in picturing this state. They did not know each other, but twice hit upon the same expression for rendering their thoughts.

One of them knows herself to be religious from the fact that she has an absolute confidence in "a supernatural power," toward which she turns in prayer "as naturally as a child turns toward its mother." She discovers religious sentiments in her heart "the whole day long," whenever she is in need of aid or comfort, in her hours of delight or sorrow, when she discovers beauty in the moral order, in nature, or in art. Her religious thoughts are characterised by a common tendency to a desire for amelioration. In the Catholic religion she finds precisely the assistance and strength which are lacking to her, the instruction that seems to her the highest to be had, and lastly the promise of a life eternal, filled with peace and light. She founds her faith and constructs her life on the notions of God, soul, and a future world. Her faith is also a source of action, and moves her to fulfil the duties of a young woman, an elder sister, and a friend; it moves her also to awaken the consoling thought of God in the hearts of those who have forgotten him; to awaken it in them who do not know it and "live on earth without light."

The other of these young women likewise declares that she believes in the supernatural, and discovers in herself religious sentiments the "whole day long," when she feels lassitude, when she looks toward "higher things," toward that which is "best" in her, when she discovers beauty, whether of the soul or of objects, for she connects everything with God "who has created all and unceasingly renews all." She distinguishes her religious thoughts by the fact that these thoughts lead her toward the good; and up to this point both our young women use the same language, revealing

thus the strong imprint of the same moral education. The latter, more intellectually inclined if I am not deceived, declares she cannot conceive of morals without religion, that she has need of a "whole, of a system that is imposed." And such is the Catholic system. Her belief in it is so necessary that if she did not think it to be absolutely valid she would "make short shrift of it" and set it aside as an obstacle if the duty which it imposed upon her was at variance with her natural tendencies. She thinks, it is true, she is impelled by her own nature to aid her neighbor, but she recognises her kindly disposition as a gift of God, and is anxious to have it serve the higher ideal which she finds in the Gospel. She wishes she had greater wealth, to do more good for the unfortunate, not only by alms but by direct personal action. It appears to her that here lies the real interest of life.

Another communication rests more upon a characteristic trait. The lady in question, who belongs to a family of high functionaries, thinks she is Catholic from the fact that she is naturally inclined to admit without examination, and to believe without discussion, anything that is taught her by recognised authority. An intense need for peace and unity holds her in an absolute faith which leaves no leeway for her exceedingly timorous and brooding spirit. She inherited from her father, who was not a believer, the habit of introspection, and this natural disposition also made confession the most necessary practice for her. Formerly, before her marriage, she became so excessively devoted to the practice as to alarm her confessor. Her religious life is richest when she feels happy and is delivered from every grave anxiety. Religious thoughts "calm" her, "fortify" her, "illuminate" her. She declares herself "essentially carried to the state of grace."

Let us sum up in a few lines the physiognomy of this interesting group.

To the question seeking to penetrate the nature of the religious sentiment, such as our correspondents could define it by self-questioning, I have received no answer up to the present in which anything else appears save the indication of some very general trait, internal peace, assurance against doubt, the aspiration toward a

higher world, filial piety, and the sentiment of the beautiful. It manifests itself now in happiness and now in misery, according to temperament, some confiding their sorrows to the Divine Master, others their joys, in order that they may be diminished or increased by his participation; for the idea of God is always present in the religious sentiment thus understood, whether it have for its object the divine will or its creation. It appears as a composite state linked even with a state of sensibility in each subject, where are also prominently marked, according to the person and occasion, the signs of fear, love, pity, æsthetic longings, or moral anxiety. The supernatural attracts the soul without merging the latter in it. The aspiration toward the divine does not reach immersion in God, which is the goal of mystical ecstasy. It does not transcend the point of orientation of desires, or of wished-for alleviation.

As for belief, it remains traditional; it is sustained by habit, and represents a sort of harmony of internal forces established by education and heredity. If doubt enters, it never eats to the foundation, but is dissipated with the enthusiasm of youth. The mind is not sufficiently original or vigorous, and life does not offer enough unexpected turns of fortune to justify it in bartering away its certitudes. The need of moral equilibrium shrinks from running the risks of reasoning. The sweetness of belief wins the day over the feeble outbursts of criticism. Reason intervenes only to give strength in its first stages, not to dispute and not to argue or examine exhaustively. The state of belief is not distinct from the emotional state.

The special notions of God, the soul, and a future world are said to be constantly present, and constitute the guides of life. The doctrine of immortality is the dearest to the heart of the woman who has suffered and wept; it would appear that she cannot tear herself from the hope of some day finding her beloved dead, illusory as the hope may be. It would appear, finally, that the religious thoughts of this class of believers impelled them to do good, to devote themselves to benefactions. And they assert this fact with so much conviction that it is necessary to see in it something more than a mere lesson learned, especially as the les-

son is one that has germinated in the rich soil of a "natural disposition." Some even cannot conceive of moral virtue outside of religion; they feel themselves or imagine themselves to be so weak that their passions would control them without this restraint.

I put no special questions regarding the worship of the Virgin and the Saints. It is remarkable that no allusion is made to them in the responses I received; but I desired the answers to be perfectly uninfluenced and took care not to indicate in any way the mode of reply. There is no doubt that many rigorous observances are practised in this group, and a more searching investigation would reveal the forms they take. But piety assumes here, upon the whole, a moderate form; it is not far-fetched, and is able to keep in harmony with the world. We have here the average class of simple and genuine Catholics. These faithful disciples, especially the women among them, do not seek the doubtful hazard of a higher theological instruction or of a thoroughgoing examination of their beliefs; they do not scrutinise the secrets of their hearts; they are docile sheep in the hands of their pastor, and they do not waste in sensual effusions or in reveries disquieting for their faith the forces which the duties of life demand of them.

III.

Our third class is perhaps less heterogeneous, and yet it is difficult to describe; the subjects that compose it revealing a species of dogmatism in which the reason and the heart appear alternately to prevail, to antagonise, and to confound each other. Here belong the majority of the priests, both secular and regular.

It is here that certain restrictions must be placed upon the value of an inquiry such as ours. In the sincerest confidences there always enters, I admit, some elements of deception; each of us unconsciously poses more or less before himself, and we accept as our own the pose that appears to us the most commendable; and the deception to which we lend ourselves denounces the quality which we do not possess, or the ideal toward which we tend. The person which we would have ourselves be thus finally conquers, or perhaps rather betrays, the person which we are.

But while this is unquestionable as regards our moral attitudes, in matters of faith it is different. Not all the persons interrogated feel themselves free to reply, and the subterfuges of those who risk replies are not apparent. But even abstention is instructive, from the manner in which it is justified; and the very trend that the thought takes in its subterfuges is also a precious datum.

A young man who was serving his novitiate with the Jesuits excused himself in the following manner to a relative who had requested him to reply to my *questionnaire*. "The majority of Catholics," he thought, "would refuse" to answer. "Upon the whole," he added, "it does not seem to me that the questions put the religious problem exactly in its right light; they virtually take it for granted that our state of soul implies nothing more than a *sentiment*, whilst it is primarily a conviction of the mind, a faith based on scientific and philosophical reasons, an intellectual doctrine antecedently to being a principle of conduct. Considered from this last point of view, our religion ought to be not the sole nor the most immediate of our reasons for acting, but it should enter and form part of all of them; in fact, in the life of a religious it is the one that most often dominates, though without excluding other reasons, etc."

We have here a characteristic trait of this new group. These people will have a religion which is a body of *knowledge*; they protest against being religious by emotion or by any unreasoned inclination of the mind. A Catholic priest, from whom I have an only too brief reply, "*distrusts*" (and I italicise the word) all the religious acts of his soul "that are expressible only as sentiments, and particularly as emotions." He knows himself to be religious by the fact that he "seeks to make sacrifices," and when "some movement of grace" directs his intelligence or his will toward God.

The word *grace* may appear here contradictory to the rational motives of criticism. It implies a singular mode of action, the value of which is more profoundly understood by the men with whom we have here to do.

The following testimonies came from two students of medicine, two clever young people thoroughly capable of introspection.

One of these young men thinks that "in its vaguest form the religious sentiment is the need of an ideal, of a belief in a world beyond, a feeling of impotence and a desire for power"; that which characterises it, he adds, and this note is here again the same, "is that it brings us near to God." He can conceive no thoughts nor religious emotions from which the idea of God is absent. Artistic emotions, sublime, philosophical thoughts, all grand and beautiful things, envelop the religious sentiment or take from it their mark, "because they are for us the point of departure of an aspiration toward God, because they are the suggestion that causes the soul to go forth from itself, etc."

These states of the religious soul may exist, according to him, without particular belief other than a belief in God and in the existence and immortality of the soul. Religion, furthermore, does not seem to him to be "a mere matter of sentiment," but a "necessity," without which life would be impossible. "As long as reason," he declares, thus emphasising the insufficiency of vague religiosity, "has no base on which to support its faith, as long as conscience has no inviolable rule by which its weaknesses are corrected, the soul will always float in dim twilight racked by uncertainty and doubt." And again, "What a transformation when faith strengthens and moulds the religious sentiment! The soul discovers the goal, the principle of all things. Its yearnings and its loves have found their object."

He attributes the defects of Catholicism to man solely. He finds in his faith "all of the precision of simple, reasonable, and immutable dogmas." His belief is also his motive of conduct. Prayer would appear to him empty were it not the principle of a "renewal" of activity. "To sacrifice everything for those who suffer, to approach sympathetically all miseries, to care for all physical and moral wounds, especially when very repulsive, to live only for the unfortunate, and in order to afford them the solace for which they look and to which they are entitled, to diffuse about one the truth of consolation and love,—such," he declares, "is what the voice of my religious conscience commands me to do."

The second of these young men, born in France of a Flemish

family, strikes a richer and more significant note. It appears that in him the religious sentiment is mingled with his innate disposition, which causes him to believe spontaneously. I discover in him a passionate imagination, an unconquerable aspiration for the Absolute, in which he shall find repose of mind and reasons for life. A peculiar sentimental substratum supports his entire intellectual edifice. As he understands it, a religion "impregnates life rather than inspires it, gives direction rather than adornment to it." He feels himself religious by the subordination of "even his future" thought, and of "even his eventual" life, to the Catholic faith, that is to say, by a conscious act of his own, although the presence of that "entirety" which is faith appears to him quite "impersonal." He admits the existence of Truth; what is more, he lives for it. "The promises of Christ," he says, "are realised in those who believe in Him; they live the life of Christ, which is to say, the Holy Spirit dwells within them."

This bent of mind is still more distinctly revealed by his classification of the methods of knowledge, of which there are two, one appertaining to the positive man and the other to the religious man. Whilst the possession of rational truths is gained singly and laboriously, the "process" by means of which he assimilated Catholicism is directly the opposite; that "cohesion of certainties" which he calls religious is for him a full guarantee of the special character of these certainties. When the Absolute comes to us, and we have recognised it, the truth which it reveals ought to be admitted in the aggregate. He recognises nothing *human* in his religion save his "act of adhesion and of love."

Thus, his belief offers the distinctly marked characteristic of being entire or of not being at all. Religion cannot be the result of human work. Revealed truths would enter necessarily even into a religion compounded of the elements of several others. But such a situation does not appear to him to be acceptable; he does not believe that we can retain the name of religion for a "partial choice of revealed truths." The certainties of religion, as he understands them, form a compact bundle, no part of which can be separated from the rest. These certainties, these "supra-sensible" truths,

needed for their acceptance merely his "yielding to the belief that the Absolute was better informed than we." The Absolute came to him and he received it.

As for discovering the Absolute in the person of Christ, our correspondent confesses that this is more difficult; but, he remarks, if we have once accepted Him as such, we must either take or reject His revelation; it is all or nothing.

He accordingly eliminates all criticism from faith, and at the same time flatters himself upon having discussed "the foundations of that authority" which he believes absolute, with all the faculties of his being, and with the free scrutiny of his conscience. Finding nothing to contradict it in science or in his own moral conscience, he is resolved not to reject one jot or tittle of it. Collectively the acts which constitute his life and which should be severally inspired by his belief, are, he avows, often in conflict with it; but they have not weakened it. One thing only would trouble his faith, and that is if opinions and acts that contravene it "were not followed with remorse."

Every religious state of soul, it appears to him, may be harmonised with some religion, the Reformation having no limits. Every man who wishes to bring his religion and his religious state of soul into accord, will find in Protestantism a free choice between the Christian truths. This is indeed an interesting reply to the question put, as to whether the religion practiced corresponded exactly to the religious state of soul, and we shall see later that the solution indicated for this fervent Catholic is actually followed by others. Nevertheless, for him no other religion is possible but his own: the Absolute could not have revealed itself in two ways.

The distinction which he makes between a philosophical system and a creed is that the former is never spontaneously admitted by us. Thus, he adds, our adhesion to a system of philosophy "never carries with it a change of heart." Thoughts, emotions, sentiments, all are religious in him, for all the forces of his being are subject to the control of faith, and tend only to "diffuse the life of grace." And it is when he believes, wishes, or feels himself performing some "apostolic act" that the religious life appears to

manifest itself in his heart most strongly. His apostleship proves his faith as generation proves life.

He speaks of the "weaknesses he has lived through." He regrets that the notions of God, the soul, and a future world are not present to him as often as he might wish. Still, the habit of meditating upon them gives to his thought a frequently unaccustomed direction. He dreads Hell as "the voluntary hatred and rejection of the love of God." He longs for Paradise "as the superior miracle of the enchantment of relative souls by absolute beauty." To introduce something of sacrifice into his life,—this is the goal, he finally avers, in which all true religion should culminate. Resignation is victory.

The religious sentiment, in fine, as we have found it in this remarkable example, does not appear to derive its sustenance in any marked degree from either fear, love, or art. It answers primarily to a disposition of the mind, the precise relations of which with the physiological tone of the subject it is difficult to discover. The religious life of our subject is centered in a sort of intellectual vision which would terminate rather in proselytic activity than in ecstasy. The apparent trait of mysticism in him is presumably his method of knowledge, an enravishment of the mind by the Absolute, which comes to him, which he recognises, and to which he abandons himself.

It would appear that his faith,—so entirely accepted that the denial of a single one of its articles would annihilate it,—remains for this very reason subject to accident and danger of dissolution; but it is also supported by the same disposition of the intellect and of the whole soul, which makes it what it is. His reason is of itself inclined to receive the miraculous assistance of grace. The remorse, in which he discovers a sort of criterion, is with him only the spontaneous reaction of a moral nature already moulded. Endowed as he is, he would never reach the calm abstention of the skeptic; he would have become a bold metaphysician if Catholicism had not subjected him to her sway, and had not disciplined at a single blow all the energies of his sentiments.

Many Catholics are akin to this type. A certain number of

women would also be grouped in this class under the head of reasoners or mystics. Among some priests, I am not warranted in saying among many, greater freedom of criticism and a more original turn of mind tends to produce a distinctively individual moral situation, where faith argues with itself and is clarified by a species of decantation. The following reply offers a significant example of this.

The ecclesiastic to whom I owe it was born and still is a Catholic. Catholicism, as he understands it, bears the entire legacy of the most distant past and the vital germ of all the civilisation of the future. Jesus was its "restorer and saviour." The Roman Church in his eyes is not Catholicism; Judæo-Alexandrian as to its dogmas and cult and Cæsarian as to its form of government, the Church is only the garment, the external and transitory form of the faith.

This priest knows himself to be religious both by his intellect and by his soul: by his intellect, because his reason transcends the finite and the sensible; by his soul, because his heart, dissatisfied with what here goes forward, hungers for the Absolute and thirsts for the Eternal.

Religious also for him is every thought, every emotion, every manifestation, every work, which binds us to the Infinite, which excites and satisfies in us this need of the divine; religious, "every man of whatever faith or dogmatism" who carries and fosters in himself this aspiration, "and who, confident of his immortality, reveals his belief in love for everything that lives and suffers about him." He sees perfectly that Catholicism "is declining and apparently perishing," in so far as it is the work of man; but he cherishes the conviction that it will assume in the future a new scientific, æsthetic, and social life, and that the gospel of the spirit will ultimately free it from the humiliating tyranny of the letter. Prayer, charity, the idea of the Beyond, the Eucharistic communion properly understood,—these are the four immovable corner-stones on which the future Church will rest: they have been disfigured, but not destroyed.

Men of this type in their higher manifestations are the light

and glory of a church; but it very frequently happens that they steal away in the night and pass to the camp of the enemy. And their flight is due to widely varying causes, more or less active according to the times and the temperament of the individual,—disaffection of the soul, the rebellion of the intellect, shattered ideals, or the source of obedience, whether it comes from the heart or the head.

But here a new chapter opens, the decomposition of belief.

IV.

We have seen faith at its heroic crisis. It holds its own here not without a struggle, and in many men and women who declare themselves to be sincere Catholics there is already marked at this stage a process of dissolution, of which I shall now indicate the principal steps.

The first situation is that of avowed Catholics, who have made a choice among the Church's beliefs and practices. We no longer find in these that steadfastness, that unity of being, which we have described. One can no longer say of such persons that they are—to borrow a phrase from one of the preceding communications—"models of their faith"; they are "reformed" without wishing it and without knowing it, and they do not wish it because they still retain at heart the Catholic soul in spite of the liberty which they allow themselves.

Such is one of our correspondents, a man of the old nobility and of a high order of merit, who knows much and has lived much. From his eighteenth to his fiftieth year he was skeptical. Some doubts, he admits, still subsist in his mind. But all his aspirations "tend toward an absolute faith." He does not rigorously observe his religious duties. His staunch faith in God nevertheless leads him to put credence in one particular doctrine,—that to which the memories of his house and family bind him. He discovers himself to be religious when in the presence of any of the manifestations of worship, and especially, although he is not of a sad turn of mind, "in mortuary ceremonies." His adhesion to the Catholic faith appears to me to be determined by a profound feeling of social

necessity, that is, by extra-religious motives. The notions of God, the soul, and future life occupy, he says, a considerable place in his reflections and intellectual occupations. He combats unceasingly the process of transformation that bids fair to overcome them. Of a combative nature, he feels himself sometimes impelled to take up arms against the attacks of free masonry on Catholicism. He detests "the men who, having in their hands the instruction of the people, aim to win them away from every belief, thus robbing them of their most efficacious weapon in the struggle for an honest life, profitable both to country and humanity."

The following are the curious statements of two women.

The first thinks that religious faith plays a considerable rôle in the inner life of woman. "If she loses," she says, "her naïve and solid faith, if dogma dissolves, on reflection, as sugar does in water, there remains nevertheless a religious instinct, a need of the ideal, a sort of instinctive certainty of a higher moral life, of an intellectual or immortal world which just as surely exists as the physical world." It is an easy thing to deny the immortality of the soul! In each one of us, she says, is a principle which connects us with the unknown God, with the eternal and immortal principle. She is not Protestant "to the letter," but she has taken her refuge in Christ. "His beautiful ethics" has conquered her forever. One of the grand disillusions of her life is her inability to practice it "in spirit and in truth," though it still remains her light and her faith.

The second of these ladies, who has responded more fully than the first, "belongs *de facto*" to the Catholic religion, but her personal tastes and predilections incline her to Protestantism. But one must, she says, be extremely religious to change one's religion, and this is not the case with her. She does not believe she has the religious spirit; she thinks that the happy ones of this world have no need of faith, which is the last resource and consolation of the unfortunate; if she seeks it and desires it without finding it, it is solely "for want of a better." She has not discovered in herself religious sentiments, save "in times of great suffering"; when pain and death have come into her life, she has "felt" the

necessity of believing in some "recompensation" in the other world, in the achievement at some future time of the destiny which we have dreamed of but not found on this earth. Her highest desire would be "to eternalise human love."

A believer to the age of fifteen, she ceased being such on reading Taine, Renan, and certain other authors. She is now twenty-five years old, and rather reverts to her faith, "but by an act of will, and from sentiment, not from reason."

She criticises the Catholic religion for its narrowness and its insufficiency; all she accepts of it is the slight vestige of Christianity which it seems to her is still left, viz., the faith in Jesus Christ. And yet she does not see in Jesus as she does in Shâkyamuni and Mohammed, only the more or less inferior manifestations of the divine thought. It is to God that everything must be traced back and to God that every rational explanation of the world, of suffering, and of life must be referred, "under penalty of death by pain and discouragement."

God and the eternity of the soul,—such are the two goals of her faith: "instinct of eternity imprisoned in a mortal body." For at bottom she has only semi-beliefs and hypotheses, in conformity to which, "lacking a better reality," she lives. It appears to her impossible to believe "without doubting what one believes." The answers of the intellect to the needs of the soul are insufficient. She does not seek to force on other persons the hypotheses that she has chosen. "Religion, faith, and love," she writes, "are individual things and belong solely to the sphere of sentiment, not to that of reason."

Yet her religion is far from being inactive. While her belief dwells within her, the power of action which she derives from it is diffused outwardly. Religious observance in her eyes is action: to do good, to suffer, to submit. Prayer is only "an aid to action, the trigger of the soul." Her religious life is recapitulated in a single phrase: love of others. Contemplation and mysticism in any of their forms seem to her not only an error but a downright fault; she sees in the world only the necessity of acting, of acting rightly, conformably to one's conscience and one's reason.

With this last lady, who appears to us as a sweet suffering soul, the religious life is especially manifested as an expression of human weakness, of a wrecked and shattered love, of a beautiful dream that has faded away. Her deeply rooted belief in the immortality of the soul and in the higher life corresponds to a logical need of moral equilibrium and of compensation, and is also the desire for an affection which shall endure beyond the tomb. Everything bordering on mysticism in the rigorous sense of an immersion of the soul in God or of a blind adhesion to faith is discarded, but not the mysticism of unfettered dreaming or vague aspiration. If the most necessary ideas are conceived as conjectures, the doubt that remains inherent in them suffers them to subsist with undiminished vivacity in those recondite recesses of conscience where reason can gain no foothold.

With this latter lady—as with the former—we have taken a forward step in the direction of the dissolution of belief. We are now passing to a second stage, which numbers of Protestants and Catholics have, in my opinion, reached,—a stage marked by a vague religiosity exhibited according to temperament in either an idealistic or mystical form. This religiosity retains nevertheless some portion of the Christian truths, and reposes on the discipline of its Church, adhering at more than one point to Christianity. But in the case of others we shall see these points of adherence weakening and loosening, until finally they vanish.

The following is the case of a lady belonging to an environment composed of jurists, physicians, and university people, whose education "has been moulded by the spirit of examination and ratiocination." At the adult age she "voluntarily" renounced Catholicism and had her children reared in the faith of "liberal Protestantism." A species of religiosity which she attributes to her conscientious submission to the commandments of the Church made its appearance in her at the time of her first communion; her gradual emancipation caused it to vanish. She does not think that she has now what is called the religious spirit, "unless," she says, "the religious spirit is nothing more than a kind of elevation toward the ideal, produced by artistic emotions, by grand spectacles

of nature, or the lofty meditations of ethics." A special belief is not necessary for her; she follows the teachings of a liberal Protestantism, which corresponds "approximately" to her religious state. She ignores "the narrow side of her creed and the few ingredients of dogma which it contains," clinging solely to its moral teachings, which she finds extremely noble.

She refrains from either denying or accepting the notions of God, the soul, and a future life; these notions, she thinks, pass the powers of human comprehension. Further, she does not deem them indispensable to action. Finally, judging from numerous examples in her own environment, she has concluded that a purely ethical instruction may suffice to form honest, upright, and high-minded beings, "whilst confessional education, pure and simple, is impotent in this regard." Catholicism, further, appears to her to exercise, in education at least, a demoralising influence. She criticises the "political spirit of the clerical party," which has so completely confounded facts, dogmas, and practices that frequently there is nothing left "among the disillusioned followers of Catholicism," of either belief, religious spirit, or morality.

A magistrate, the husband of the lady just cited, also suddenly renounced during adolescence the doctrines of the Roman Church. Later, his thought, "enlightened by observation of the social conduct of Catholics," led him "to repudiate absolutely traditional religion and even to look upon the Catholic spirit as detrimental to genuine devotion and to the following of a satisfactory moral code." He can now only call himself a free-thinker; he "dares to affirm" that he is so without intolerance. He does not believe himself religious; he does not see the least necessity for a particular belief of a dogmatic character, and abides by the conception of a body of moral duties, "or if the expression appear more appropriate, by a body of obligations of conscience toward oneself and others." He does not reject as void of all interest "the problems relating to the existence of a personal God, of an individual immaterial principle surviving the body of each one of us, and called the soul"; but he declares himself powerless to find the solution of them and does not believe that the rules of social conduct are in need of a doctrine

founded on these mysterious questions. He inclines rather "to a sort of pantheism."

To the same class belongs a Parisian lady, wealthy, intelligent, infirm, and blighted in her dearest affection. She abandoned completely the Catholic religion, of which even the ceremonies scarcely impress her, so greatly is she irritated by "their underlying fatuity." She follows no creed, accepts neither revelation nor supernaturalism, and declares herself refractory in metaphysics, although idealistic in spite of this fact. "Gazing on the starry heavens, or on some magnificent spectacle of nature," she writes, "I am profoundly troubled, and I avow that I comprehend with difficulty that intelligent beings can find full satisfaction in artistic admiration only. Before the infinitely small I experience the same ecstasy .... These are the only moments when I can discern in myself a religious sentiment, if I may at all so qualify this emotion." Common sense, she thinks, dictates that in the lack of a religious belief she should abstain from curious inquiries; but, though ignorant of science, she has the "speculative" spirit, and these lofty questions "fascinate her nevertheless."

She is in no need of a particular belief, although she would prefer not "to live in the open air"; in following the good she is at least sure of not being deceived. She feels within herself an ideal of moral beauty and of kindness, in communion with which she loves to live, but which she grieves to see contravened by the brutal laws of nature. Her intellectual peace is troubled by it, even when her conscience is at rest.

Never in the most sorrowful hours of her life has she felt the need of turning toward God, who, even if he existed, would not, she thinks, interest himself in her individuality. Her consolation has always come from her sentiment of duty, from comparison of her own state with that of those more unfortunate than herself, from her resigned submission to the laws of the universe; for she believes that there are laws in the moral order just as there are in the physical. The word *God* corresponds to so gigantic an interrogation point for her that she abhors all cynical and blasphemous spirits of negation. She does not know whether he exists, nor

whether the soul exists; she has the impression that our soul is extinguished with death. These ideas regarding human destiny suffice for the conduct of her life; but she is averse nevertheless to disseminating them, since she thinks them insufficient from a social point of view.

Another lady less favored by fortune and who appears to be very reticent, refuses to explain herself at length. "One can," she writes, "easily confess things to oneself which one would not like to commit to others." Further, it would be somewhat like criticising those who had reared her and also those with whom she associated; again, "since all things hang together" (I draw attention to this passage), "and since the portion of sentiment with which every one is burdened never remains inactive, it would also be necessary to say what use she had made of her portion; and why it tended to direct her to one side rather than to another." She affirms, nevertheless, that she is absolutely lacking in religiosity. "Far from its being a privation to me," she writes with some warmth, "I believe that its consoling feature is the conviction that there is nothing after death, that one has made an end of all suffering. Every one puts his hope where he can; mine asks nothing but oblivion, that is in the other world, for in this I have other pretensions."

A young woman of twenty-six, an Angevin, wealthy, very intellectual, and of good family, gives us in compensation a very extended communication, interesting in all points. She was a believer and Catholic up to her fourteenth year. From the day of her first communion she ceased to be such. "I divined," she writes, "the lies, the crafty machinations, the selfish policy; I fathomed the consciencelessness, the despotism, and especially the slavery, of fervent Catholics." Out of regard for her family, and from a sort of intellectual laziness and a cowardly respect for public opinion, she nevertheless followed the practices of her religion. But she reproached herself for it, and especially she reproached Catholicism for its antagonism of the Gospel, on which it rested, and for not admitting freedom of will and individual control of conscience. For these reasons the Protestant religion was

more to her taste. The latter at least did not enforce such puerile practices and did not put love under the tutelage of the confessor, and "banish beauty."

Such is the dominating note of this character which appears to me to be essentially artistic, lofty, and with a passion for excessive imagination.

The thoughts and the emotions which she would characterise as religious, "or rather as sacred," spring up in her when contemplating some beautiful spectacle of nature, or on hearing some beautiful work of man. "Sometimes," she says, "on the summit of a mountain, in the evening, and in the neighborhood of glaciers, I have had for the moment the conviction that some power, some reason, some higher superhuman beneficence, presided over so much harmony and order; but I have never likened this powerful, intelligent, beneficent, and *divine* being, so vaguely and fugitively conceived, to the God that the modern Catholic Church portrays." When she sees the gloom created about her by the priests, she ardently longs for the "æsthetic morals of the Athenians, their light-hearted gladness, and the humanity of the Greek cults in the fourth century before Christ."

The dominating duty in her eyes is respect for oneself and one's ideals, the employment of one's faculties "in life for life." She "is absolutely in doubt regarding the serious and fundamental questions of future life, eternity, our ultimate destiny, and to what is commonly called our *soul*. None of the solutions proposed satisfy her reason; she thinks that we know nothing, that we are sure of nothing, and that a vast amount of vanity, credulity, stupidity, superficiality, or lunacy, is requisite to assert that either God exists or does not exist, and to draw any conclusions from such a hypothesis."

A very significant communication comes finally, from another young woman, a Parisian and a composer of merit, distinguished in every respect. Reared by non-communicating parents, she belongs to no religion. She boldly declares being unable to accept even the principle of God, that is, "the existence of a supreme Force acting consciously on what we perceive of the universe."

She can see no argument deduced from the beauty, duration, or regularity of our world that affirms this principle. She does not admire the world, "being unable to compare it to anything else but itself." She is unable, therefore, to follow any of the existing religions; being only tentative explanations of the unknown, to her it seems natural that they should change, seeing that they depend, like every hypothesis, on the number and the state of the things known at the time of their formation.

But if she deems herself incompetent to judge the world, nothing prevents her, by way of compensation, she says, from loving it. She loves it, then, "as the fish loves the water, because it cannot live without it." She cannot see that this felicitous harmony which keeps us in accord with things is even a subject for admiration, which is an emotion rather than a judgment. She seeks to reduce everything to the intellectualistic point of view,—the divine principle as well as religion and ethics. But her sentiment resumes its rights, and she describes wonderfully the state of her soul, which is highly remarkable.

Unbelieving as she might at first appear and coldly rationalistic, she yet discovers that the religious sentiment is highly developed in her. Her sympathy extends to those who believe; she understands them and they understand her. There is nothing in their lives of which she does not seize instinctively the most delicate manifestation. "Yes," she declares, "all who have ever believed are my equals, my brothers, whilst nothing is more foreign to me than the incredulous common sense that does not dream. The accents, the expressions, of their faith penetrate me with so profound and genuine an emotion that I have been compelled more than once to give voice to my feelings in music." And an artist "in her language never lies."

"It is odd," she continues, with her refreshing candor, "that my heart can be stirred by what has never troubled my reason, but it is a fact, and I have long fostered without knowing it the contradiction that abides in me."

She partly, though unwittingly, explains this contradiction when she supposes the probable reason of it is that "religions are

the most beautiful and touching creations of man." Or again, when she says that the religious sentiment "resides" in every being who feels that ideas are of life and that "religion must be the imperfect and variable expression of a sentiment which exists before any defined belief, and itself creates its own object."

The fact is that her creative nature protests both against irreligion and against traditional dogmatism, against the barrenness of heart inherent in negation and the curbing of fancy by the letter. She has need of faith and of freedom. The same stroke of the wing that exalts the great religious mystics above the poverty of vulgar religion carries her beyond precisely defined dogmas and narrow observances and causes her to accept no more of religion than the aspiration for the beautiful, which is the divine, and the sincerity of effort to attain it. Her intellectual deficiencies are offset by her spontaneous artistic genius, and we have here a striking example of the resurrection of the religious sentiment by art, a proof of the conciliating truth that all noble creatures are at bottom acting by desire and by living belief.

In our group of dissidents of varying degrees we have hitherto met with the partial rejection or negation of Catholicism without any clearly conceived doctrine being instituted to take its place. The last stage is marked by the deliberate substitution of one belief for another, of a coherent body of ideas and sentiments for ideas and sentiments that have been undermined. This substitution is in some instances effected either by the adoption of a doctrine already formed, like positivism, or by the constant employment of one's forces in the search for some new truth. Examples of this class abound, but one should be careful not to imagine that this is the common practice, or that disintegration of the Christian faith (for Christianity at large is essentially involved here) necessarily carries with it in all cases a distinct advantage. The ruin of traditional ideas is only too often synonymous with a reversion to an inferior state, not only in the ignorant classes, but even in those which possess the polish of a higher culture.

I have unqualifiedly denounced the coarse devotions rampant in the Catholic religion; it has preserved the use of the amulet,

and its belief in miraculous interventions has led to the most puerile practices. But religion alone, I must reiterate, is not involved here; the ancient and vital character of man comes to light in these miseries. A like inferiority is discoverable in many men who have abandoned religion, and we have found at the limit of the dissolution the same ethical and moral situation that stamps its lower grades. We have seen quite recently all manner of people thronging about the doors of ignorant women necromancers, and the less disguised humbuggery of occultism never wants its patrons.

Simple and straightforward souls in whom faith vanishes are more apt to revert to a kind of natural religion. Such is the case of an honest workwoman of the environs of Paris. To the question whether she believed in God, she replied: "He has never had much concern for me in my times of trouble. Why should I concern myself about him?" To the question whether God existed, she answered that she knew nothing of the kind, and she thought with all the other people round about that the real god was the sun which made everything grow. To the other question, whether she expected nothing after death, whether all ended with death, she said: "We are like the plants; they spring up and they die, and then new ones spring up." She had no time to go to church, but she would not be averse to spending an hour there on Sunday, for it was "a tidy place" where one could rest and hear good songs and music. More people than we are aware of in our agricultural districts in France have reverted to this childish paganism. The situation is, relatively speaking, favorable, since there comes to light in it at least a need of poetry and a desire for something higher. But how many people fall to a still lower state, to the lowest level of the animal! They have lost utterly the little decency which religion, despite all, infuses into life at some hours. They seek for nothing else in this world but enjoyment and the indulgence of their lusts.

V.

The idolatrous mass which composed our first group are not followers of French Protestantism, which is the result of a selec-

tion. The Reformation could not reach this stagnating residuum of the population, these ethnical offscourings massed together in certain remote corners of our provinces, whose superstitions the Catholic priests have purified and sometimes utilised when unable to eradicate them. The very rejection of a constant and miraculous divine intervention in the pettiest affairs of life carries with it among Protestants important consequences in belief and religious observances. Perhaps we are constrained to see here a really fundamental difference; and the acceptance of miracles, however limited, nevertheless stamps with a definite mark Catholics of the higher order, akin as they may be otherwise to Protestant Christians of the same class.

We should doubtless have discovered signs of decomposition, or rather, of the transformation of belief in certain Catholic priests who have remained nevertheless good Catholics,—could we have obtained their confidences. Similar indications distinctly appear in some Protestants. M. W. Monod[1] has recently told us of the doubts which the orthodox theology of Calvinism inspired in Vinet. Not only would Vinet not admit the plenary inspiration of the Scriptures, and not only did he regard the Bible as a collection of human writings, but he was troubled throughout his whole life by the difficulties which the dogma of redemption offered to his mind; he could not conceive of the idea of substitution which served as the basis of this dogma and which appeared to him contradicted by our moral notions.

M. Auguste Sabatier[2] was seized with similar doubts. He discarded both the primitive theological notion of a ransom paid by God to Satan and the consequent juridical conception of an objective satisfaction rendered to God under the form of a debt paid to a creditor or of a substitutive penalty agreed on by the judge; he sees in the drama of Calvary only the most sublime example among many others of the devotion of love,—condition of salvation and of new life in the moral world.

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[1] *Vinet douteur* (Paris, Fischbacher).

[2] *Études de théologie et d'histoire* (Paris, Fischbacher).

Our correspondents, as we have seen, do not enter into these discussions. The dissolution of belief is effected, among the greatest number of them, by the way of vague reasoning. When doubt has forced a French Catholic out of the church, he only rarely takes refuge in Protestantism, and, if at all, only for a short time; usually he overleaps it also with the same bound, and this is one of the reasons why we cannot believe in the success of Protestantism in France, at least in the form in which we see it to-day.

As to the Jews, my personal observations on their religious mentality has been confirmed in various ways. A lady who kindly volunteered to obtain responses to my *questionnaire* writes: "I have not been successful with the most intelligent Jews, who have confessed to me that they no longer believe in their religion, but do not care to say anything derogatory of it; this is a general sentiment in Israel."

These reservations being made, let us attempt to draw some conclusions from the psychological data presented, directing our attention chiefly to the three questions of religious sentiment, faith or religious conviction, and the notions of "God" and "soul."

It would be too hazardous to affirm that the religious sentiment is weakening or for that matter that it was preserving its full vitality in society. The external observation of a cult does not necessarily imply a vigorous piety, and piety also may be met with apart from any definite religious practice. I am at least inclined to think that the diseased form of religious sentiment which finds expression in asceticism with its extreme consequences now appears less frequently. Thus, there is most likely being effected in the heart of Catholicism a transformation which has already begun in Protestantism.

Honor is paid to Luther for having dissipated the chimera of an angelic life superior to the ordinary life of men, and for having shown that piety dwells in the accomplishment of the humblest duties, social life being of greater value than solitude, marriage than celibacy, the full development of our faculties than their repression. And this was "the most absolute contradiction of every-

thing that the Church had lived, believed, and practised up to this time.[1]

Protestantism profited thus by the natural progress which granted to new societies a less troubled and more solidly established existence, but this influence also made itself felt in the moral life of Catholicism. The opposition became still more pronounced between the two currents which have always carried its advancement, and the celebrated quarrel in which Bossuet arose against Fénelon contributed to the triumph of the piety that set its foundation in reason and action.

To every effort to establish a religion based on an intellectualistic psychology, there corresponds, it is true, a reaction. We see it to-day take the form of a vague religiosity and a delicate mysticism culminating in a sort of neo-Christianity fostered by a sickly and impassioned sensibility. This sensibility, furthermore, does not find its sole expression in the form of religious mysticism; it extends over all of life, and we see in it a disposition inherent in human nature which produces certain manifestations of saintliness, as it has produced others in politics, art, and philosophy.

We must distinguish, therefore, between what is healthy and durable and what is sickly and perishable; and in my opinion it is the healthy form of religious sentiment that tends to prevail, although it is impossible to say at present to what end the existing crisis in belief is to lead, or what power of reality the religious sentiment is to have in the absence of a positive faith and of every poetical myth, especially when bereft of that communion of the soul with God which appears after all to be its ground and necessary sustenance.

Are not the great artists of religion, and often too the great toilers in practical religious life, virtual mystics? They strive to curb their natural inclinations, to abandon themselves to the divine will, and to unite their soul with God, using to this end the forces which other men expend on the ordinary labors and enjoyments of life. Whatever passions human beings possess they consume in

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[1] Félix Kuhn, *Le christianisme de Luther* (Paris, Fischbacher).

the accomplishment of their moral ideals, we know not at the price of what inward struggles and physical tribulations.

But is it not true that religion loses its color and warmth when we pass from sentiment to intellect, from the heart to the brain? History and observation prove it. It is not, therefore, indifferent to the success of a religion,—this waning of the mystical disposition, or rather this diversion, notably for the benefit of art, of that strange energy springing from the sources of our being which the goal of saintship or proselytism formerly claimed entire.

Further, the decline of mysticism goes hand in hand with the ruin of the mystical method of knowledge. Scientific discipline is gradually gaining the upper hand, being demanded by conditions of work in modern societies. The possibility is now less easily conceived of two methods of knowledge, of which one consists in the mysterious operation of grace, or more simply in deductions *a priori*. A process of intuition and pure logic can scarcely be imagined which should give results acceptable in the face of the facts of positive and objective experience; and this question pushed to its limits is perhaps the critical case of metaphysics, if not also of philosophy. But let us take care not to decide this grave question; the religious fact, intuition is a fact, a valid and undeniable experience.

Ordinary believers, it is true, are not burdened with grave anxieties of this sort; their convictions, born of habit and tranquil apathy, do not appear to me to have either gained or lost in power. Their belief remains what it has always been, a body of vague ideas in which the notions of God and soul alone assume prominence; although even these do not often or efficaciously influence their conduct.

The dearest of all to the hearts of women is the belief in immortality, and it is this same hope of a future life that gains for spiritism so many followers. Among philosophers themselves it is known that some have gone so far as to retain the doctrine of personal survival whilst abandoning or diminishing the importance of the idea of God. But in this last respect I am struck with the great difference of the situation between the French Catholic and

the American Protestant, such as Professor Leuba has portrayed the latter in his inquiry.[1]

A striking change is indicated by him in the affective relations of man to the divinity. "Freedom and equality," he says, "are in modern society the key-note of the relation of the individual to his fellow-men; it seems to be becoming also the key-note of his relations to God. The feeling of freedom towards God has largely supplanted the duty-motive which the Catechisms continue to inculcate: action in obedience to God's command is out of fashion; it is what is right and what is best which is now the favorite reason for conduct. Even a certain feeling of equality, monstrous as this may seem, has passed into the attitude of the people to God: awe, reverence, worship, appear only dimly and not as frequently as is assumed, in the religious consciousness of the democratic Anglo-Saxon."

The case is certainly different with the French Catholic. He hopes for nothing from God save by submission, and he never seeks to force the divine assistance save by prayer. A "sentiment of impotence" which has doubtless also its dignity causes him to bend his knee and holds him in the hands of his Master, who is his stay and his compassion.

Among many persons who have remained believers, we have noticed an entire abandonment of religious practices. The usual cause of their disaffection is the weak personality of certain preachers or monks, sometimes also the attitude of the clergy and their political machinations. I know many pious persons, and even clergymen, who are foremost in denouncing the puerile devotions, greed, commercialism, and misconduct so frequently laid to the charge of the clergy; and we have here one of the main reasons of the hostility that turns from the attack of the unworthy or incompetent servitors of religion to the attack of religion itself. The severity of the public judgment is guaged by the importance of the rôle played, and the gravity of the deception is measured by the object of the calling.

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[1] See *The Monist*, July, 1901.

As to the moral effects of religious teaching, the reader will doubtless have made the observation in reading so many pages filled with good intentions that life would be incomparably better and ethics higher in religious circles if the acts conformed at all closely to the intentions; but this is not the case, and many of our believers are subject to illusions on this point. The idea which they have of good conduct, even when they do not realise it fully, may nevertheless restrain them from falling to a lower level. Several, we have seen, have the feeling that their passions would overpower them if they were not checked by their belief. They undoubtedly slander themselves and are seeking for an argument in favor of an ethics founded on their religion; but their confession is sincere in many cases, and the moralists who deny to religious education so powerful an influence for good, ought at least to accord to it the merit of preventing some evil.

To sum up, what we ordinarily find is the constancy of faith received in childhood, and the return to that faith when shattered for a time or neglected in the course of life.

Among so many examples, I may be permitted to cite that of a master of narration who recently died.[1] His end forms a strange contrast with the spirit of a Parisian journalist, skeptic, and scoffer. When he felt his end approaching, he returned to the religious faith and practices of his early years; every night he repeated his prayers before the curate of his parish, whom he had summoned. One of his old comrades[2] having come to visit him, he said: "Old fellow, I thank you for having come; I press your hand for the last time." And when his friend sought to comfort him, he continued: "Do not seek to deceive me. *My sacrifice has been made.*" Words profoundly Christian and Catholic, remarked his friend to me,—"words which I heard pronounced by my mother and my father on their death-bed. How they impressed me coming from his mouth, me who have known him for thirty years!"

We have here a man reared in the Catholic faith. This man

[1] Aurelian Scholl.

[2] M. le Marquis de Charnacé, who told me the story of this last visit and has permitted me to use his name here.

left his native province, entered the paths of journalism, and became a hero of the boulevard. He wrote of everything, but probed no question to the bottom. Everything religious remained indifferent to him as long as he enjoyed his youth and his health and could indulge in unlimited hopes. The fatal hour sounded at last; then the memories of his childhood and of forgotten sentiments came to life again. The metaphysical animal, always concealed deep within every one of us, awakened in him; the dream of his artificial life was dissipated; the two ends of his existence met, over and above the futile agitation that had filled it.

Things are different with the man who has lived a life of serious thought and ripe deliberation, or with one who has not received a religious education in childhood. Such a one, Littré for example, may in his last hours make a concession to his family but not be *converted* in the real meaning of the term. What we do not find, or very seldom find, is the return to Catholicism of a mind which has been long emancipated, or the conversion of a non-believer.

The same remark applies to Protestantism, and we may see in these facts an indication for the future. Statistics do not inform us regarding the process of the dissolution of belief, of which we have gathered such striking testimonies; let us be on our guard, therefore, against supposing that this process has not its limit or its offset, and to conclude without further reflection that its effects are always beneficial.

We have seen that there exists with us, as with all nations, a mass of men and women having superstitions without true faith and observing the rules of their creeds without participation of the heart, a mass docile to all influences and ready to fall to the lowest plane the moment they are not supported by habit and by the commandments imposed upon them. It would be as dangerous to withdraw from this class their support as it would be foolhardy to affect disdain for the ancient beliefs which satisfied so many lofty and noble souls.

That which ought to perish will perish. Time will always accomplish the ruin which our hand is so eager to inaugurate. Violence never leads to success in the battle of religious ideas; liberty

of conscience is a better guarantee of peace than force, and individuals always know how to find ingenious compromises when a partisan law would embroil everything.

It is not without dismay that I hear philosophers condemning the despotism of morals and of the family and proclaiming the ruin of the ideas in which so many find repose; or authors and playwriters presenting to their public personalities bereft of every notion of duty and of every serious character in life. Instead of such a state of mind representing the expansion and liberation of the individual, I fear it marks its deterioration and servitude. I see what is sought to be eradicated from the modern soul, but I do not see what can be added to it for its enrichment. Everything that hitherto constituted the grandeur of man,—sacrifice, devotion, and heroism,—is supposed to serve no purpose. The sentiments that we prized so highly,—filial piety, respect, virtue, and patriotism,—are declared to be illusory and perishable. We imagine abrogated at a blow and by a miracle of evolution the soul which evolution itself has created. We transport ourselves bodily to an age in the future in which all things of to-day—institutions and even persons—shall no longer be what they now are, in which everything shall be changed,—thoughts and habits, passions and desires. We suffer ourselves to be enravished by the mirage of an approaching age of gold, just as blind and uncritical as the ideologues and verbose poets of the centuries past. We are again reconstructing on paper the world and life, with never a thought for present living humanity.

I do not wish to say that there is nothing sound and acceptable in the criticisms and ambitions of philosophers. The world is involved in continual change. The true meaning of the modifications that are going forward must be apprehended so that we may lend to them the right assistance. It is an ancient proverb that we destroy only what we replace. It is no longer the right of any one to control by his individual action the movement of ideas and facts. The task of comprehending them is in itself difficult enough, and each one of us may find in this field subject for ample reflection.

PARIS. LUCIEN ARRÉAT.

THE PHILOSOPHICAL FOUNDATIONS OF MATHEMATICS.

I. HISTORICAL INTRODUCTION.

MATHEMATICS as commonly taught in our schools is based upon axioms. These axioms so called are a few simple formulas which the beginner must take on trust.

Axioms are defined to be self-evident propositions, and are claimed to be neither demonstrable nor in need of a demonstration. They are statements which are said to command the assent of every one who comprehends their meaning.

Euclid does not use the term "axiom." He starts with Definitions (ὅροι), which describe the meanings of point, line, surface, plane, angle, etc. He then propoṣes Postulates (αἰτήματα) in which he takes for granted that we can draw straight lines from any point to any other point, and that we can prolong any straight line in a straight direction. Finally, he adds what he calls Common Notions (κοιναὶ ἔννοιαι) which declare that things which are equal to the same thing are equal to one another; that if equals be added to equals, the wholes are equal, etc.

I need not mention here that the readings of the several manuscripts vary, and that some propositions (e. g., that all right angles are equal to one another) are now missing, now counted among the postulates, and now adduced as common notions.

In our modern editions of Euclid we find a statement concerning parallel lines added either to the Postulates or Common Notions. Originally it appeared in Proposition 29 where it is needed to prop up the argument that proves the equality of alternate angles on parallels, viz., that—

"those straight lines which, with another straight line falling upon them, make the interior angles on the same side less than two right angles, do meet if continually produced."

This is exactly the point to be proved, for it is only one form of the conception of parallelism. So it was formulated axiomatically as follows:

"If a straight line meet two straight lines, so as to make the two interior angles on the same side of it taken together less than two right angles, these straight lines being continually produced, shall at length meet upon that side on which are the angles which are less than two right angles."

This dogmatic statement (now commonly called "the axiom of parallels") was naturally transferred by the editors of Euclid to the introductory portion of the book where it now appears either as the fifth Postulate or the eleventh, twelfth, or thirteenth Common Notion. The latter is obviously the less appropriate place, for the idea of parallelism is assuredly not a Common Notion; it is not a rule of pure reason such as would be an essential condition of all thinking, reasoning, or logical argument. And if we do not give it a place of its own, it should either be classed among the postulates, or recast so as to become a pure definition.

It seems to me that no one can read the axiom of parallels as it stands in Euclid without receiving the impression that the statement was affixed by a later redactor. Even in Proposition 29, the original place of its insertion, it comes in as an afterthought; and if Euclid himself had considered the difficulty of the parallel axiom, so called, he would have placed it among the postulates in the first edition of his book, or formulated it as a definition.[1]

Though the axiom of parallels must be an interpolation, it is of classical origin, for it was known even to Proclus (410–485 A. D.), the oldest commentator of Euclid.

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[1] For Professor Halsted's ingenious interpretation of the origin of the parallel theorem see *The Monist*, Vol. IV., No. 4, p. 487. He believes that Euclid anticipated metageometry, but it is not probable that the man who wrote the argument in Proposition 29 had the fifth Postulate before him. He would have referred to it or stated it at least approximately in the same words. But the argument in Proposition 29 differs considerably from the parallel axiom itself.

By the irony of fate, the theory of the parallel axiom, the authorship of which is extremely doubtful, has become more closely associated with Euclid's name than anything he has actually written, and when we now speak of Euclidean geometry we mean a system based upon this conception of parallelism.

The word axiom (ἀξίωμα) means "honor, reputation, high rank, authority," and is used by Aristotle, almost in the modern sense of the term, as "a self-evident highest principle," or "a truth so obvious as to be in no need of proof." It is connected with the verb ἀξιοῦν, "to deem worthy, to think fit, to maintain," and with ἄξιος, "worth" or "worthy."

The commentators of Euclid who did not understand the difference between Postulates and Common Notions, spoke of both as axioms, and even to-day the term Common Notion is mostly so translated.

We may state here at once that all the attempts made to derive the axiom of parallels from pure reason were necessarily futile, for no one can prove the absolute straightness of lines, or the evenness of space, by logical argument. Therefore these concepts, including the theory concerning parallels, cannot be derived from pure reason; they are not Common Notions and possess a character of their own. But the statement seemed thus to hang in the air, and there appeared the possibility of a geometry, and even of several geometries, in whose domains the parallel axiom would not hold good. This large field has been called metageometry, hypergeometry, or pangeometry, and may be regarded as due to a generalisation of the space-conception involving what might be called a metaphysics of mathematics.

Mathematics is a most conservative science. Its system is so rigid and all the details of geometrical demonstration are so complete, that the science was commonly regarded as a model of perfection. Thus the philosophy of mathematics remained undeveloped almost two thousand years. Not that there were not great mathematicians, giants of thought, men like the Bernouillis, Leibnitz and Newton, Euler, and others, worthy to be named in one breath with Archimedes, Pythagoras, and Euclid, but they ab-

stained from entering into philosophical speculations, and the very idea of a pangeometry remained foreign to them. They may privately have reflected on the subject, but they did not give utterance to their thoughts, at least they left no records of them to posterity.

It would be wrong, however, to assume that the mathematicians of former ages were not conscious of the difficulty. They always felt that there was a flaw in the Euclidean foundation of geometry, but they were satisfied to supply any need of basic principles in the shape of axioms, and it has become quite customary (I might almost say orthodox) to say that mathematics is based upon axioms. In fact, people enjoyed the idea that mathematics, the most lucid of all the sciences, was at bottom as mysterious as the most mystical dogmas of religious faith.

Metageometry has occupied a peculiar position among mathematicians as well as with the public at large. The mystic hailed the idea of n-dimensional spaces of curvature and other concepts of which we can form definite concepts but can have no concrete representations. They promised to justify all his speculations and give ample room for innumerable notions that otherwise would be doomed to irrationality. In a word, metageometry has always proved attractive to erratic minds. Among the professional mathematicians, however, those who were averse to philosophical speculation looked upon it with deep distrust, and therefore either avoided it altogether or rewarded its labors with bitter sarcasm. Prominent mathematicians did not dare to risk their reputation, and consequently many valuable thoughts must have remained unpublished. Even Gauss did not dare to speak out boldly, but communicated his thoughts to his most intimate friends under the seal of secrecy, not unlike a religious teacher who fears the odor of heresy. He did not mean to suppress his thoughts but he wanted to bring them before the public only in their mature shape. A letter to Taurinus concludes with the remark:

"Of a man who has proved himself a thinking mathematician, I fear not that he will misunderstand what I say, but under all circumstances you have to regard it merely as a private communication of which in no wise public use, or one that

might lead to it, is to be made. Perhaps I shall publish them myself in the future if I should gain more leisure than my circumstances at present permit."

C. F. GAUSS.

GÖTTINGEN, 8. November, 1824.

The first attempt at dispensing with the problem of parallelisms was made by Nasir Eddin (1201–1274) whose work on Euclid was printed in Arabic in 1594 in Rome. His labors were taken up by John Wallis who in 1651 communicated Nasir Eddin's expositions of the fifth Postulate in a Latin translation to the mathematicians of the University of Oxford, and then propounded his own views in a lecture delivered on July 11, 1663. Nasir Eddin takes his stand upon the postulate that two straight lines which cut a third straight line, the one at right angles, the other at some other angle, will converge on the side where the angle is acute and diverge where it is obtuse. John Wallis, in his endeavor to prove this postulate, starts with the auxiliary theorem:

> If a limited straight line which lies upon an unlimited straight line be prolonged in a straight direction, its prolongation will fall upon the unlimited straight line.

There is no need of entering into the details of his proof. We may call his method the proposition of the straight line and will grant him that he proves the straightness of the straight line. In his further argument John Wallis shows the close connection of the problem of parallels with the notion of similitude. If we grant the one we must accept the other, and thus in Euclidean geometry the theorem holds good, for of any figure we can construct similar figures of any size.

Girolamo Saccheri, a learned Jesuit of the seventeenth century,[1] attacked the problem in a new way. He saw the close connection of parallelism with the right angle, and in his work on Eu-

[1] Born Sept. 5, 1667, in San Remo. He became a member of the Jesuit order March 24, 1685, and served as a teacher of grammar at the Jesuit College di Brera, in Milan, his mathematical colleague being Tommaso Ceva (a brother of the more famous Giovanni Ceva). Later on he became Professor of Philosophy and Polemic Theology at Turin and in 1697 at Pavia. He died in the College di Brera Oct. 25, 1733.

clid[1] proposed three possibilities. If in the quadrilateral figure $ABCD$ the sides $AB = CD$ and the angles in A and B are right angles, the angles at C and D may be acute or obtuse. He undertakes to prove the absurdity of these assumptions so as to leave the only solution the sole possibility left, viz., that they must be right angles. But he finds difficulty in pointing out the contradiction to which these assumptions may lead and thus he opens a path on which Lobatchevsky (1793–1856) and Bolyai (1802–1860) followed, reaching a new view which makes three geometries possible, viz., the geometries of (1) the acute angle, (2) the obtuse angle, and (3) the right angle, the latter being the Euclidean geometry, in which the theorem of parallels holds.

While Saccheri seeks the difficulty of the problem in the notion of the right angle in the special case of a quadrangle, the German mathematician Lambert[2] starts from the notion of the sum of the angles of a triangle being equal to 180 degrees. There are three possibilities: the sum may be exactly equal to, more than, or less than 180 degrees. The first will make the triangle a figure in a plane, the second renders it spherical, and the third produces a geometry on the surface of an imaginary sphere. As to the last hypothesis Lambert said not without humor:[3]

"This result[4] possesses something attractive which easily suggests the wish that the third hypothesis might be true."

He then adds:[5]

"But I do not wish it in spite of these advantages, because there would be innumerable other inconveniencies. The trigonometrical tables would become in-

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[1] *Euclides ab omni naevo vindicatus; sive conatus geometricus quo stabiliuntur prima ipsa universae geometriae principia.* Auctore Hieronymo Saccherio Societatis Jesu. Mediolani, 1773.

[2] Johann Heinrich Lambert was born August 26, 1728, in Mühlhausen, a city which at that time was a part of Switzerland. He died in 1777. His *Theory of the Parallel Lines*, written in 1766, was not published till 1786, nine years after his death, by Bernoulli and Hindenburg in the *Magazin für die reine und angewandte Mathematik.*

[3] P. 351, last line in the *Mag. für die reine und angewandte Math.*, 1786.

[4] That if in the square $ABCD$, the angle in G were less than 90 degrees (Lambert says 80 degrees), the side AB would be equal to AD.

[5] *Ibid.*, p. 352.

finitely more complicated and the similitude as well as proportionality of figures would cease altogether. No figure could be represented except in its own absolute size; and astronomy would be in a bad plight, etc."

Lobatchevsky's geometry is an elaboration of Lambert's third hypothesis and it has been called "imaginary geometry" because its trigonometric formulas are those of the spherical triangle if its sides are imaginary, or, as Wolfgang Bolyai has shown, if the radius of the sphere is assumed to be imaginary $=\sqrt{-1}$.

Lambert lays much stress on the proposition that each of the three hypotheses must be regarded as valid if only one case of it be found to be valid.

France has contributed least to the literature on the subject. Augustus de Morgan records the following story concerning the efforts of her greatest mathematician to solve the Euclidean problem. Lagrange, he says, composed at the close of his life a discourse on parallel lines. He began to read it in the Academy but suddenly stopped short and said: "Il faut que j'y songe encore." With these words he pocketed his papers and never recurred to the subject.

Legendre's treatment of the subject appears in the third edition of his elements of Euclid, but he omitted it from later editions as too difficult for beginners. Like Lambert he takes his stand upon the notion of the sum of the angles of a triangle, and like Wallis he relies upon the idea of similitude, saying that "the length of the units of measurement is indifferent for proving the theorems in question."[1]

A new epoch begins with Gauss, or rather with his ingenious disciple Riemann. While Gauss was rather timid about speaking openly on the subject, he did not wish his ideas to be lost to posterity. In a letter to Schumacher, dated May 17, 1831, he said:

"I have begun to jot down something of my own meditations, which are partly older than forty years, but which I have never written out, being obliged therefore to excogitate many things three or four times over. I do not wish them to pass away with me."

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[1] *Mémoires de l'Académie des Sciences de l'Institut de France.* Vol. XII. 1833.

The notes to which Gauss here refers have not been found among his posthumous papers, and it therefore seems probable that they are lost and our knowledge of his thoughts remains limited to the comments that are scattered through his correspondence with mathematical friends.

Gauss wrote Bessel (1784–1846) January 27, 1829:

"I have also in my leisure hours frequently reflected upon another problem, now of nearly forty years' standing. I refer to the foundations of geometry. I do not know whether I have ever mentioned to you my views on this matter. My meditations here also have taken more definite shape, and my conviction that we cannot thoroughly demonstrate geometry *a priori* is, if possible, more strongly confirmed than ever. But it will take a long time for me to bring myself to the point of working out and making public my *very extensive* investigations on this subject, and possibly this will not be done during my life, inasmuch as I stand in dread of the clamors of the Bœotians, which would be certain to arise, if I should ever give *full* expression to my views. It is curious that *in addition to* the celebrated flaw in Euclid's Geometry, which mathematicians have hitherto endeavored in vain to patch and never will succeed, there is still another blotch in its fabric to which, so far as I know, attention has never yet been called and which it will by no means be easy, if at all possible, to remove. This is the definition of a plane as a surface in which a straight line joining *any two* points lies *wholly* in that plane. This definition contains *more* than is requisite to the determination of a surface, and tacitly involves a theorem which is in need of prior proof."

Bessel in his answer to Gauss makes a distinction between Euclidean geometry as practical and metageometry (the one that does not depend upon the theorem of parallel lines) as true geometry. He writes under the date of February 10, 1829:

"I should regard it as a great misfortune if you were to allow yourself to be deterred by the 'clamors of the Bœotians' from explaining your views of geometry. From what Lambert has said and Schweikardt orally communicated, it has become clear to me that our geometry is incomplete and stands in need of a correction which is hypothetical and which vanishes when the sum of the angles of a plane triangle is equal to 180°. This would be the *true* geometry and the Euclidean the *practical*, at least for figures on the earth."

In another letter to Bessel, April 9, 1830, Gauss sums up his views as follows:

"The ease with which you have assimilated my notions of geometry has been a source of genuine delight to me, especially as so few possess a natural bent for

them. I am profoundly convinced that the theory of space occupies an entirely different position with regard to our knowledge *a priori* from that of the theory of numbers (*Grössenlehre*); that perfect conviction of the necessity and therefore the absolute truth which is characteristic of the latter is totally wanting to our knowledge of the former. We must confess in all humility that a number is *solely* a product of our mind. Space, on the other hand, possesses also a reality outside of our mind, the laws of which we cannot fully prescribe *a priori*."

Another letter of Gauss may be quoted here in full. It is a reply to Taurinus and contains an appreciation of his essay on the Parallel Lines. Gauss writes from Göttingen, Nov. 8, 1824:

"Your esteemed communication of October 30th, with the accompanying little essay, I have read with considerable pleasure, the more so as I usually find no trace whatever of real geometrical talent in the majority of the people who offer new contributions to the so-called theory of parallel lines.

"With regard to your effort, I have nothing (or not much) more to say, except that it is incomplete. Your presentation of the demonstration that the sum of the three angles of a plane triangle cannot be greater than 180°, does indeed leave something to be desired in point of geometrical precision. But this could be supplied, and there is no doubt that the impossibility in question admits of the most rigorous demonstration. But the case is quite different with the second part, viz., that the sum of the angles cannot be smaller than 180°; this is the real difficulty, the rock on which all endeavors are wrecked. I surmise that you have not employed yourself long with this subject. I have pondered it for more than thirty years, and I do not believe that anyone could have concerned himself more exhaustively with this second part than I, although I have not published anything on this subject. The assumption that the sum of the three angles is smaller than 180° leads to a new geometry entirely different from our Euclidean,—a geometry which is throughout consistent with itself, and which I have elaborated in a manner entirely satisfactory to myself, so that I can solve every problem in it with the exception of the determining of a constant, which is not *a priori* obtainable. The larger this constant is taken, the nearer we approach the Euclidean geometry, and an infinitely large value will make the two coincident. The propositions of this geometry appear partly paradoxical and absurd to the uninitiated, but on closer and calmer consideration it will be found that they contain in them absolutely nothing that is impossible. Thus, the three angles of a triangle, for example, can be made as small as we will, provided the sides can be taken large enough; whilst the area of a triangle, however great the sides may be taken, can never exceed a definite limit, nay, can never once reach it. All my endeavors to discover contradictions or inconsistencies in this non-Euclidean geometry have been in vain, and the only thing in it that conflicts with our reason is the fact that if it were true there would necessarily exist in space a linear magnitude quite *determinate in it-*

self, yet unknown to us. But I opine that, despite the empty word-wisdom of the metaphysicians, in reality we know little or nothing of the true nature of space, so much so that we are not at liberty to characterise as *absolutely impossible* things that strike us as unnatural. If the non-Euclidean geometry were the true geometry, and the constant in a certain ratio to such magnitudes as lie within the reach of our measurements on the earth and in the heavens, it could be determined *a posteriori*. I have, therefore, in jest frequently expressed the desire that the Euclidean geometry should not be the true geometry, because in that event we should have an absolute measure *a priori*."

Schweikart, a contemporary of Gauss, may incidentally be mentioned as having worked out a geometry that would be independent of the Euclidean axiom. He called it astral geometry.[1]

Gauss's ideas fell upon good soil in his disciple Riemann (1826–1866) whose Habilitation Lecture on "The Hypotheses which Constitute the Bases of Geometry" inaugurates a new epoch in the history of the philosophy of mathematics.

Riemann states the situation as follows. I quote from Clifford's almost too literal translation (first published in *Nature*, 1873):

"It is known that geometry assumes, as things given, both the notion of space and the first principles of constructions in space. She gives definitions of them which are merely nominal, while the true determinations appear in the form of axioms. The relation of these assumptions remains consequently in darkness; we neither perceive whether and how far their connection is necessary, nor, *a priori*, whether it is possible.

"From Euclid to Legendre (to name the most famous of modern reforming geometers) this darkness was cleared up neither by mathematicians nor by such philosophers as concerned themselves with it."

Riemann arrives at a conclusion which is negative. He says:

"The propositions of geometry cannot be derived from general notions of magnitude, but the properties which distinguish space from other conceivable triply extended magnitudes are only to be deduced from experience."

In the attempt at discovering the simplest matters of fact from which the measure-relations of space may be determined, Riemann declares that—

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[1] *Die Theorie der Parallellinien, nebst dem Vorschlag ihrer Verbannung aus der Geometrie.* Leipzig und Jena. 1807.

"Like all matters of fact, they are not necessary, but only of empirical certainty; they are hypotheses."

Being a mathematician, Riemann is naturally bent on deductive reasoning, and in trying to find a foothold in the emptiness of pure abstraction he starts with general notions. He argues that position must be determined by measuring quantities, and this necessitates the assumption that length of lines is independent of position. Then he starts with the notion of manifoldness, which he undertakes to specialise. This specialisation, however, may be done in various ways. It may be continuous, as is geometrical space, or consist of discrete units, as do arithmetical numbers. We may construct manifoldnesses of one, two, three, or n dimensions, and the elements of which a system is constructed may be functions which undergo an infinitesimal displacement expressible by dx. Thus, spaces become possible in which the directest linear functions (analogues to the straight lines of Euclid) cease to be straight and suffer a continuous deflection which may be positive or negative, increasing or decreasing.

Riemann argues that the simplest case will be, if the differential line-element ds is the square root of an always positive integral homogeneous function of the second order of the quantities dx in which the coefficients are continuous functions of the quantities x, viz., $ds = \sqrt{\Sigma dx^2}$, but it is one instance only of a whole class of possibilities. He says:

"Manifoldnesses in which, as in the Plane and in Space, the line-element may be reduced to the form $\sqrt{\Sigma dx^2}$, are therefore only a particular case of the manifoldnesses to be here investigated; they require a special name, and therefore these manifoldnesses in which the square of the line-element may be expressed as the sum of the squares of complete differentials I will call *flat*."

The Euclidean plane is the best-known instance of flat space being a manifold of a zero curvature.

Flat space has also been called by the new-fangled word *homaloidal* (from ὁμαλός, level), which recommends itself as a term in distinction from the popular meaning of even.

In applying his determination of the general notion of a manifold to actual space, Riemann expresses its properties thus:

"In the extension of space-construction to the infinitely great, we must distinguish between *unboundedness* and *infinite extent*; the former belongs to the extent-relations, the latter to the measure-relations. That space as an unbounded threefold manifoldness, is an assumption which is developed by every conception of the outer world; according to which every instant the region of real perception is completed and the possible positions of a sought object are constructed, and which by these applications is forever confirming itself. The unboundedness of space possesses in this way a greater empirical certainty than any external experience. But its infinite extent by no means follows from this; on the other hand, if we assume independence of bodies from position, and therefore ascribe to space constant curvature, it must necessarily be finite, provided this curvature has ever so small a positive value. If we prolong all the geodesics starting in a given surface-element, we should obtain an unbounded surface of constant curvature, i. e., a surface which in a *flat* manifoldness of three dimensions would take the form of a sphere, and consequently be finite."

It is obvious from these quotations that Riemann is a disciple of Kant. He was inspired mainly by his teacher Gauss and by Herbart. But while he starts a transcendentalist, employing mainly the method of deductive reasoning, he arrives at results which would stamp him as an empiricist of the school of Mill. He concludes that the nature of real space, which is only one instance among many possibilities, must be determined *a posteriori*. The problem of tridimensionality and homaloidality are questions which must be decided by experience, and while upon the whole he seems inclined to grant that Euclidean geometry is the most practical for a solution of the coarsest investigations, he is inclined to believe that real space is non-Euclidean. Though the derivation from the Euclidean standard can only be slight, there is a possibility of determining it by exact measurement and observation.

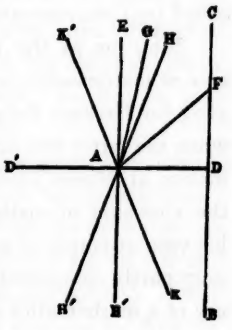
Riemann has succeeded in impressing his view upon metageometers down to the present day. They have built higher and introduced new ideas, yet the corner-stone of metageometry remained the same. It will therefore be found recommendable in a discussion of the problem to begin with a criticism of his Habilitation Lecture.

It is regrettable that Riemann was not allowed to work out his philosophy of mathematics. He died at the premature age of forty, but his work was continued by two other friends of Gauss,

Lobatchevsky and Bolyai, who actually contrived a geometry independent of the theorem of parallels.

It is perhaps no accident that the two independent and almost simultaneous inventors of a non-Euclidean geometry are original, not to say wayward, characters living on the outskirts of European civilisation, the one a Russian, the other a Magyar.

Nicolai Ivanovich Lobatchevsky[1] was born Oct. 22 (Nov. 2 of our calender), 1793, in the town of Makariev, about 40 miles above Nijni Novgorod on the Volga. His father was an architect; at his death in 1797 his widow and two small sons were left in poverty. At the gymnasium Lobatchevsky was noted for his wilfulness and disobedience, and he escaped expulsion only through the protection of his mathematical teacher, Professor Bartels, who even then recognised the extraordinary talents of the boy. Lobatchevsky graduated with distinction and became in his further career professor of mathematics and in 1827 Rector of the University of Kasan. Two books of his offered for official publication were rejected by the paternal government of Russia, and the manuscripts may be considered as lost for good. Of his several essays on the theories of parallel lines we mention only the one which made him famous throughout the whole mathematical world, *Geometrical Researches on the Theory of Parallels.*[2]



Lobatchevsky divides all lines, which in a plane go out from a point *A* with reference to a given straight line *BC* in the same plane, into two classes—cutting and not cutting. In progressing from the not-cutting lines, such as *EA* and *GA*, to the cutting lines, such as *FA*, we must come upon one *HA* that is the boundary between the two classes; and it is this which he calls the par-

[1] The name is spelled differently according to the different methods of transcribing Russian characters.

[2] For further details see Prof. G. B. Halsted's article "Lobachévski" in *The Open Court*, 1898, pp. 411 ff.

allel line. He designates the parallel angle on the perpendicular ($p = AD$, dropped from A upon BC) by Π. If $\Pi(p) < \frac{1}{2}\pi$ (viz., 90 degrees) we shall have on the other side of p another angle $DAK = \Pi(p)$ parallel to DB, so that on this assumption we must make a distinction of *sides in parallelism*, and we must allow two parallels, one on the one and one on the other side. If $\Pi(p) = \frac{1}{2}\pi$ we have only intersecting lines and one parallel; but if $\Pi(p) < \frac{1}{2}\pi$ we have two parallel lines as boundaries between the intersecting and non-intersecting lines.

We need not further develop Lobatchevsky's idea. Among other things, he proves that "if in any rectilinear triangle the sum of the three angles is equal to two right angles, so is this also the case for every other triangle," that is to say, each instance is a sample of the whole, and if one case is established, the nature of the whole system to which it belongs is determined.

The importance of Lobatchevsky's discovery consists in the fact that the assumption of a geometry from which the parallel axiom is rejected, does not lead to self-contradictions but to the conception of a general geometry of which the Euclidean is one possibility. This general geometry was later on most appropriately called by Lobatchevsky "Pangeometry."

John (or, as the Hungarians say, János) Bolyai imbibed the love of mathematics in his father's house. He was the son of Wolfgang (or Farkas) Bolyai, a fellow student of Gauss at Göttingen when the latter was nineteen years old and was professor of mathematics at Maros Vásárhely. Farkas wrote a two-volume book on the elements of mathematics[1] and in it he incidentally mentions his vain attempts at proving the axiom of parallels. His book was only partly completed when his son János wrote him of his discovery of a mathematics of pure space. He said:

"As soon as I have put it into order, I intend to write and if possible to publish a work on parallels. At this moment, it is not yet finished, but the way which I have followed promises me with certainty the attainment of my aim, if it is at all attainable. It is not yet attained, but I have discovered such magnificent things

[1] *Tentamen juventutem studiosam in elementa matheos* etc. *introducendi*, printed in Maros Vásárhely.

that I am myself astonished at the result. It would forever be a pity, if they were lost. When you see them, my father, you yourself will concede it. Now I cannot say more, only so much that *from nothing I have created another wholly new world.* All that I have hitherto sent you compares to it as a house of cards to a castle."[1]

János being convinced of the futility of proving Euclid's axiom, constructed a geometry of absolute space which would be independent of the axiom of parallels. And he succeeded. He called it the *Science Absolute of Space*,[2] an essay of twenty-four pages which Bolyai's father incorporated in the first volume of his *Tentamen* as an appendix.

Bolyai was a thorough Magyar. He was wont to dress in high boots, short wide Hungarian trousers, and a white jacket. He loved the violin and was a good shot. While serving as an officer in the Austrian army, János was known for his hot temper, which finally forced him to resign his commission as a captain, and we learn from Halsted that for some provocation he was challenged by thirteen cavalry officers at once. János calmly accepted and proposed to fight them all, one after the other, on condition that he be permitted after each duel to play a piece on his violin. We know not the nature of these duels nor the construction of the pistols, but the fact remains assured that he came out unhurt. As for the rest of the report that "he came out victor from the thirteen duels, leaving his thirteen adversaries on the square," we may be permitted to express a mild but deep-seated doubt.

János Bolyai starts with straight lines in the same plane, which may or may not cut each other. Now there are two possibilities: there may be a system in which straight lines can be drawn which do not cut one another, and another in which they all cut one another. The former, the Euclidean, he calls Σ, the latter S. "All theorems," he says, "which are not expressly asserted for Σ or for S are enunciated absolutely, that is, they are true whether Σ or S is

[1] See Halsted's introduction to the English translation of Bolyai's *Science Absolute of Space*, p. xxvii.

[2] *Appendix scientiam spatii absolute veram exhibens: a veritate aut falsitate axiomatis XI. Euclidei (a priori haud unquam decidenda) independentem; Adjecta ad casum falsitatis quadratura circuli geometrica.*

reality."[1] The system S can be established without axioms and is actualised in spherical trigonometry (*ibid. p.* 21). Now S can be changed to Σ, viz., plane geometry, by reducing the constant i to its limit (where the sect $y=0$) which is practically the same as the construction of a circle with $r=\infty$, thus changing its periphery into a straight line.

The labors of Lobatchevski and Bolyai are significant in so far as they prove beyond the shadow of a doubt that a construction of geometries other than Euclidean is possible and that it involves us in no absurdities or contradictions. This upset the traditional trust in Euclidean geometry as absolute truth, and it opened at the same time a vista of new problems, foremost among which was the question as to the mutual relation of these three different geometries.

It was Cayley who proposed an answer which was further elaborated by Felix Klein. These two ingenious mathematicians succeeded in deriving by projection all three systems from one common aboriginal form called by Klein *Grundgebild* or the Absolute. In addition to the three geometries hitherto known to mathematicians, Klein added a fourth one which he calls *elliptic*.[2]

Thus we may now regard all the different geometries as three species of one and the same genus and we have at least the satisfaction of knowing that there is *terra firma* at the bottom of our mathematics, though it lies deeper than was formerly supposed.

Prof. Simon Newcomb of Johns Hopkins University, although not familiar with Klein's essays, worked along the same line and arrived at similar results in his article on "Elementary Theorems Relating to the Geometry of a Space of Three Dimensions and of Uniform Positive Curvature in the Fourth Dimension."[3]

In the meantime the problem of geometry became interesting to outsiders also, for the theorem of parallel lines is a problem of

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[1] See Halsted's translation, p. 14.

[2] "Ueber die sogenannte nicht-euklidische Geometrie" in *Math. Annalen*, 4, 6 (1871–1872). *Vorlesungen über nicht-euklidische Geometrie*, Göttingen. 1893.

[3] Crelle's *Journal für die reine und angewandte Mathematik*. 1877.

space. The best treatment of the subject came from the pen of the great naturalist Helmholtz whose two essays are so interesting because written in a most popular style.[1]

A collection of all the materials from Euclid to Gauss, compiled by Paul Stäckel and Friedrich Engel under the title *Die Theorie der Parallellinien von Euklid bis auf Gauss, eine Urkundensammlung zur Vorgeschichte der nicht-euklidischen Geometrie*, is perhaps the most useful and important publication in this line of thought, a book which has become indispensable to the student of metageometry and its history.

A store of information may be derived from Bertrand A. W. Russell's essay on the *Foundations of Geometry*. He divides the history of metageometry into three periods: The synthetic, consisting of suggestions made by Legendre and Gauss; the metrical, inaugurated by Riemann and characterised by Lobatchevsky and Bolyai; and the projective, represented by Cayley and Klein, who reduce metrical properties to projection and thus show that Euclidean and non-Euclidean systems may result from "the absolute."

Among modern writers no one has done more in the interest of metageometry than the indefatigable Dr. George Bruce Halsted, Professor of Mathematics in the University of Texas.[2] He has not only translated Bolyai and Lobatchevsky, but has also offered his own theories toward the solution of the problem.

Prof. B. J. Delbœuf and Prof. H. Poincaré have expressed their conceptions as to the nature of the bases of mathematics, in

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[1] "Ueber die thatsächlichen Grundlagen der Geometrie," in *Wissenschaftl. Abh.*, 1866, Vol. II., p. 610 ff., and "Ueber die Thatsachen, die der Geometrie zum Grunde liegen," *ibid.*, 1868, p. 618 ff.

[2] From among his various publications we mention only his translations: *Geometrical Researches on the Theory of Parallels by Nicholaus Lobatchewsky.* Translated from the Original. And (2) *The Science Absolute of Space, Independent of the Truth or Falsity of Euclid's Axiom XI. (which can never be decided a priori). By John Bolyai.* Translated from the Latin, both published in Austin, Texas, the translator's present place of residence. Further, we refer the reader to Halsted's bibliography of the literature on hyperspace and non-Euclidean Geometry in the *American Journal of Mathematics*, Vol. I., pp. 261–276, 384, 385, and Vol. II., pp. 65–70.

articles contributed to *The Monist*.[1] The latter treats the subject from a purely mathematical standpoint, while Dr. Ernst Mach in his essay "On Physiological, as Distinguished from Geometrical, Space" (published in *The Monist* for April, 1901) attacks the problem in a very original manner and takes into consideration mainly the natural growth of space conception. His exposition might be called "the physics of geometry."

I cannot conclude this essay without paying a tribute to the memory of Hermann Grassmann of Stettin, a mathematician of first degree whose highly important results in this line of work have only of late found the recognition which they so fully deserve. I do not hesitate to say that Hermann Grassmann's *Lineare Ausdehnungslehre* is the best work on the philosophical foundation of mathematics from the standpoint of a mathematician.

Grassmann establishes first the idea of mathematics as the science of pure form. He shows that the mathematician starts from definitions. He then proceeds to show how the product of thought may originate either by the single act of creation, or by the double act of positing and combining. The former is the continuous form, or magnitude, in the narrower sense of the term, the latter the discrete form or the method of combination. He distinguishes between intensive and extensive magnitude and chooses as the best example of the latter the sect[2] or limited straight line. Hence the name of the new science, "theory of linear extension."

Grassmann constructs linear formations of which systems of one, two, three, and n degrees are possible. The Euclidean plane is a system of second degree, and space a system of third degree, He thus generalises the idea of mathematics, and having created a

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[1] They are as follows: "Are the Dimensions of the Physical World Absolute?" by Prof. B. J. Delbœuf, *The Monist*, January, 1894; "On the Foundations of Geometry," by Prof. H. Poincaré, *The Monist*, October, 1898; also "Relations Between Experimental and Mathematical Physics," *The Monist*, July, 1902.

[2] Grassmann's term is *Strecke*, a word connected with the Anglo-Saxon "Stretch," being that portion of a line that stretches between two points. The translation "sect," than which there is no better, was suggested by Prof. G. B. Halsted.

science of pure form, points out that geometry is one of its applications which originates under definite conditions. Grassmann made the straight line the basis of his geometrical definitions. He defines the plane as the totality of parallels which cut a straight line and space as the totality of parallels which cut the plane. Here is the limit to geometrical construction, but abstract thought knows of no bounds. Having generalised our mathematical notions as systems of first, second, and third degree, we can continue in the numeral series and construct systems of four, five, and still higher degrees. Further, we can determine any plane by any three points, given in the figures x_1, x_2, x_3, not lying in a straight line. If the equation between these three figures be homogeneous, the totality of all points that correspond to it will be a system of second degree. If this homogeneous equation is of the first grade, this system of second degree will be simple, viz., of a straight line; but if the equation be of a higher grade, we shall have curves for which not all the laws of plane geometry hold good. The same considerations lead to a distinction between homaloidal space and non-Euclidean systems.[1]

Being professor at a German gymnasium and not at a university, Grassmann's book remained neglected and the newness of his methods prevented superficial readers from appreciating the sweeping significance of his propositions. Since there was no call whatever for the book, the publishers returned the whole edition to the paper mill, and the complimentary copies which the author had sent out to his friends are perhaps the sole portion that was saved from the general doom.

Grassmann, disappointed in his mathematical labors, had in the mean time turned to other studies and gained the honorary doctorate of the University of Tübingen in recognition of his meritorious work on the St. Petersburg Sanskrit Dictionary, when Victor Schlegel called attention to the similarity of Hamilton's theory of vectors to Grassmann's concept of *Strecke*, both being limited straight lines of definite direction. Suddenly a demand for Grass-

[1] See Grassmann's *Ausdehnungslehre* von 1844, Anhang I., pp. 273–274.

mann's book was created in the market; but alas! no copy could be had, and the publishers deemed it advisable to reprint the destroyed edition of 1844. The satisfaction of this late recognition was the last joy that brightened the eve of Grassmann's life. He wrote the introduction and an appendix to the second edition of his *Lineare Ausdehnungslehre*, but died while the forms of his book were on the press.

At the present day the literature on metageometrical subjects has grown to such an extent that we do not venture to enter into further details. We will only mention the appearance of Professor Schoute's work on more-dimensional geometry[1] which promises to be the elaboration of the pangeometrical ideal.

Having reviewed all the attempts made to establish non-Euclidean systems of geometry, it may not be out of place to state, that in spite of the well-deserved fame of the metageometricians from Wallis to Halsted, Euclid's claim to classicism remains unshaken. The metageometrical movement is not a revolution against Euclid's authority but an attempt at widening our mathematical horizon. Let us hear what Halsted, one of the boldest and keenest heroes among the champions of metageometry of the present day, has to say of Euclid. Halsted begins the Introduction to his English translation of *Bolyai's Science Absolute of Space* with a terse description of the history of his great book *The Elements of Geometry*, the rediscovery of which is not the least factor that initiated a new epoch in the development of Europe which may be called the era of inventions, of discoveries, and of the appreciation as well as growth of science. Halsted says:

"The immortal *Elements* of Euclid was already in dim antiquity a classic, regarded as absolutely perfect, valid without restriction.

"Elementary geometry was for two thousand years as stationary, as fixed, as peculiarly Greek, as the Parthenon. On this foundation pure science rose in Archimedes, in Apollonius, in Pappus; struggled in Theon, in Hypatia; declined in Proclus; fell into the long decadence of the Dark Ages.

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[1] *Mehrdimensionale Geometrie* von Dr. P. H. Schoute, Professor der Math. an d. Reichs-Universität zu Groningen, Holland. Leipzig, Göschen's Verlag. So far only the first volume, which treats of linear space, has appeared.

"The book that monkish Europe could no longer understand was then taught in Arabic by Saracen and Moor in the Universities of Bagdad and Cordova.

"To bring the light, after weary, stupid centuries, to western Christendom, an Englishman, Adelhard of Bath, journeys, to learn Arabic, through Asia Minor, through Egypt, back to Spain. Disguised as a Mohammedan student, he got into Cordova about 1120, obtained a Moorish copy of Euclid's *Elements*, and made a translation from the Arabic into Latin.

"The first printed edition of Euclid, published in Venice in 1482, was a Latin version from the Arabic. The translation into Latin from the Greek, made by Zamberti from a MS. of Theon's revision, was first published at Venice in 1505.

"Twenty-eight years later appeared the *editio princeps* in Greek, published at Basle in 1533 by John Hervagius, edited by Simon Grynaeus. This was for a century and three-quarters the only printed Greek text of all the books, and from it the first English translation (1570) was made by 'Henricus Billingsley,' afterward Sir Henry Billingsley, Lord Mayor of London in 1591.

"And even to-day, 1895, in the vast system of examinations carried out by the British Government, by Oxford, and by Cambridge, no proof of a theorem in geometry will be accepted which infringes Euclid's sequence of propositions.

"Nor is the work unworthy of this extraordinary immortality.

"Says Clifford: 'This book has been for nearly twenty-two centuries the encouragement and guide of that scientific thought which is one thing with the progress of man from a worse to a better state.

"'The encouragement; for it contained a body of knowledge that was really known and could be relied on.

"'The guide; for the aim of every student of every subject was to bring his knowledge of that subject into a form as perfect as that which geometry had attained.'"

Euclid's *Elements of Geometry* are not counted among the books of divine revelation, but truly they deserve to be held in religious veneration. There is a sanctity in mathematical truth which is not sufficiently appreciated, and certainly if truth, helpfulness, and directness and simplicity of presentation, give a title to rank as divinely inspired literature, Euclid's great work should be counted among the canonical books of mankind.

* * *

The author has purposely introduced what might be called a personal element in these expositions of a subject which is commonly regarded as dry and abstruse, and endeavored to give something of the lives of the men who have struggled and labored

in this line of thought and have sacrificed their time and energy on the altar of one of the noblest aspirations of man, the delineation of a philosophy of mathematics.

There is one more point which should be mentioned in this connection. It is the close analogy which mathematics bears to religion. We have first the rigid presentation of mathematical truth discovered (as it were) by instinct, by a prophetic divination, for practical purposes, in the shape of a dogma as based upon axioms, which is followed by a period of unrest, being the search for a philosophical basis, which finally leads to a higher standpoint from which, though it acknowledges the relativity of the primitive dogmatism, consists in a recognition of the eternal verities on which are based all our thinking, and being, and yearning.

Is there any need of warning our readers that this sketch of the history of metageometry is both brief and popular? We have purposely avoided the discussion of technical details, limiting our exposition to the most essential points and trying to show them in a light that will render them interesting to the non-mathematical reader also. The article is intended to serve as an introduction to a ventilation of the problem itself, and the author proposes to continue the subject in a discussion anent the foundation of mathematics—an article which he hopes to be able to submit to his readers in a subsequent number of *The Monist*. He has abstained from criticising the views of his predecessors because he wishes first to submit his own solution to the criticism of professional mathematicians.

EDITOR.

PRIMITIVE THEORIES OF KNOWLEDGE: A STUDY IN LINGUISTIC PSYCHOLOGY.

INSPECTION of the words which, in various languages, express the idea "to know," reveals the fact that, with different peoples, these have sprung from different roots. In other words, there have been, as there are now, several primitive theories of knowledge, each basing itself upon one of the senses in particular, to the greater or less exclusion of some or of all of the rest.

The Zapotecs of the state of Oaxaca, whose ancestors were one of the half-civilised nations of old Mexico, had a very simple theory of knowledge. In their language, Dr. Brinton (*Amer. Antiq.*, 1893, p. 382) tells us: "The verb 'to know' is a reduplication of the first person of the personal pronoun, *na*, 'I'; *na-na*, 'to know,' literally, 'my mine,' 'that which is with me, essentially mine' It is absolutely the same in the Huasteca dialect of the Mayan stock." The name *Nahuatl* (the plural *Nahua* is applied to the people themselves), by which the language of ancient Mexico is known, signifying "able, skilful, superior," is derived, according to the same authority, from "the monosyllable *na*, which, in several closely related linguistic stocks in Southern Mexico, means 'to know, knowledge.'"

Dr. Brinton points out the resemblance in these tongues between the word for "knowledge, know," and the pronoun of the first person, observing further: "The man possessed of knowledge is everywhere he who is able. He *can* because he *kens*. There is no need to teach the world the Baconian maxim: 'Knowledge is power'; it is always recognised." In our own language *can*, *ken*, *know*, are all closely related, but are not exactly the same word. *Can* and *know* (with German *können*, Latin *gnoscere*, Greek γιγνώσκειν, Sanskrit *jnâ*) go back to a primitive Indo-European root *gen* (*gne*, *gno*), "to recognise, know," which by some authorities is thought to be kin with the radical *gen*, "to produce, bear," but the seeming identity may be quite accidental. *Ken*, cognate with German *kennen*, has come into English from the Scandinavian, and is really a causal verb signifying originally "to make to know," "to show," "to teach." From the root *gen* come ultimately the German *Kunst*, "art" (in the older language also "wisdom, skill"); the Greek γνῶσις, "knowledge"; the Latin *notion* (whence our "notion"), "making acquainted" (then "idea"); English *cunning*, "knowledge, skill" (now com-

monly with an implication of deceit or underhandedness), borrowed from the Icelandic. As Trench has pointed out (*Select Gloss.*, 1879, p. 63), the sense of "crooked knowledge" (Bacon already wrote: "We take *cunning* for a sinister or crooked wisdom") is not the earliest English meaning of the term, for Fox, in his *Book of Martyrs*, published in 1562, could say of the persons of the Trinity "all these three Persons are even in power, and in *cunning*, and in might." The modern American application of *cunning* to pictures, babies, birds, trees, houses, flowers, teeth, anything indeed, is a notional *tour de force*, a linguistic triumph, although it seems to have been largely anticipated in most of its senses by the writers of the Elizabethan era in England.

From the same Indo-European root *gen* is derived the Teutonic *kóni*, "knowing, wise," which, with some changes in sound and signification, appears in English *keen*, "sharp, eager, acute"; Icelandic *kœnn*, "wise, experienced"; German *kühn*, "bold." The original implication of *keen* may have been "wise or able in battle," "skilled in the use of war-weapons," since, as Kluge (*Etym. Wörterb.*, 1894, p. 220) remarks, many of the intellectual and moral ideas of the primitive Teutons are based upon the experiences of war and combat,—out of the struggle for existence has grown the knowledge of men. Knowledge born of trial the Latins called *experientia*, "what one has gone through." Goethe, the great German poet, crystallised the old race-thought, when he declared that "experience is the only genuine knowledge." Science too, to-day, is genuine through experiment. We *know* only after we have *done*, or, at least, *tried*, *experimented*. The Greek ἐμπειρία (whence our *empiricism*), "experience, knowledge," comes from πεῖρα, "trial." The word *science*, through the Latin *scientia*, "knowledge, skill, expertness, science," comes from the verb *scire*, "to know (in the widest sense), understand, be skilled in." The primitive meaning of *scire* seems to have been "to distinguish, to discern,"—more primitively still, "to split, divide."

Back to the same Indo-European root we can trace our English word *skill*. Philosophy, as a branch of human knowledge, thus comes quite naturally by its "hair-splitting" proclivities. From a similar radical, with like meaning, are derived the Latin *certus* (whence our "certain," etc.), *cernere*, "to separate, distinguish, perceive, decide" (whence our "discern"), etc. The German *gescheit*, "wise, clever, sensible," is derived ultimately from the Indo-Germanic root *skhaid*, "to split, separate," and even to-day "*splitting* the difference" is often the cleverest way out of a dispute.

When we speak of "a *sapient* young man," "a *sapient* remark," "the Divine *Sapience*," we do not always remember the primitive meaning of these words. But the cognate *sapid*, *insipid*, give us the clue to their origin. The Latin *sapere* (whence the French *savoir*), before it came to signify "to be prudent, discreet, wise," "to know, understand," meant "to taste, savor, smack." It is an interesting fact that the French, who are pre-eminently the people of "taste," should have selected this particular Latin word as their general term for "to know." In Eng-

lish we still say "a *smack* of knowledge." The *sage*, for the word comes from the Latin *sapius*, is a man of "taste," therefore, as the *wise* man is one of "sight." It is worth remembering also that it was after "*tasting* of the fruit of the tree of *knowledge*," according to the Hebrew Scriptures, that the first human beings became as gods "*knowing* good and evil."

Sprachgefühl still makes us speak, with unconscious correctness, of "the *sagacity* of a hound." The words *sagacity* and *sagacious*, which connect themselves with the Latin adjective *săgax* ("wise, sagacious, keen, acute, shrewd"; but, more primitively, "keen or quick scented") and the verb *săgire* ("to perceive keenly by the senses, perceive, see acutely"; primitively "to scent out") are really smell-terms. The original sense of *săgire* seems to have been "to scent out, trace, track"; and from the same old Aryan root *săg* have been developed English *seek*, German *suchen* (Gothic *sokjan*) and their cognates. Here the knower is "the seeker after," "the scenter out," just as to-day among the Zulus of South Africa, the primitive sage or shaman is "the *smeller out* of witches." To the savage mind even in the world beyond the grave the final test of knowledge is sometimes *smell*. With the inhabitants of certain districts of Borneo the practice prevails of whispering a message into the ear of the dead, and M. Hose (*Geogr. Journ.*, 1893, p. 198) informs us: "Upon his [the funeral was that of a boy] arrival in Appo Leggan (i. e., Hades) they would probably greet him in terms such as these: 'O Grandchild, it was for you the gong was beating, which we heard just now. What have you brought? How are they all up above? Have they sent any messages?' The new arrival then delivers the messages entrusted to him, and gives the cigarettes [which, rolled up in a piece of banana-leaf, have been placed in his hand] as proof of the truth of what he says. These cigarettes retain the *smell* of the hands which made them, which the dead relations are able to recognise."

Dr. D. G. Brinton (*Pursuit of Happiness*, 1893, p. 132) calls attention to the fact that of old the pleasures of the sense of smell "were considered the most delightful of all, even to Divinity itself." Hence the "sweet savor" which Jahveh liked to smell, the "odor of sanctity," etc., besides the countless metaphors and similes, alike in literature and in folk-thought in which the sense of smell or the nose figures as the representative of what is pleasantly perceived or known. Our words *smell* and *perfume* literally "end in *smoke*,"—for "smoke" is their ultimate signification. Curiously enough, *Geruch*, the German word for the sense of smell, has a similar origin, going back, with *riechen* ("to smell"), our English *reek*, and the German substantive *Rauch*, "smoke," to a Teutonic radical *ruk*, whose primitive meaning is "to smoke." The German word for "taste," *Geschmack*, is of mixed physiological and psychological heredity,—the Middle High German *smac* signified "taste, smell, scent."

The English *scent* is derived from Latin *sentire*, "to feel, perceive by the senses, hear, see, smell, taste,"—a word of broad meaning psychically and physio-

logically. Of like broad significance is the Latin *sensus*, whence our *sense*, with its cognates in other languages, borrowing from the Latin or Romance tongues *Sentire*, as Kluge notes, is related to the Irish *sêt*, "way, path," the Latins having given to the idea of "feeling one's way" (as we even yet term it in English) an abstract turn. *Sinn*, the German word for "sense" has a like origin. Together with *sinnen*, which in Old High German had the meaning "to travel, strive to go,"—possibly the later meaning of the word has been influenced by the Latin *sensus*,—the word *Sinn* comes from the Teutonic root *sintha* ("road, journey"), seen in Gothic *sinths* and cognate with Irish *sét*, all of which, with the Latin *sentire*, go back to a primitive Indo-European radical, of which the branches are *sent-* "to direct towards, think of," *sento*, "path, way," and *sen*, "to go, travel, strive, think."

The idea of *thought* as *travel* is known to not a few primitive peoples, and modern *telepathy* revives the old conception. When we say "there is no royal *road* to learning," we are repeating an ancient story. The "*avenues* of the senses" are older than physiological psychology. In the religions and philosophies of the world, "ways" have been very prominent. We have the "Royal Path" and the "Heavenly Path" of Confucius, the *Taou* or "Way" of Laou-Tze, the "Noble Path" ("Four Paths") of Buddhism, the "Way" of Christianity, etc. Moreover, the *Pontiff* of the Roman Church, as his Latin name, *pontifex*, indicates, had a predecessor, who was simply "the pathmaker." πεῖρα, the root of the Greek ἐμπειρία, "experience, knowledge," is connected with πόρος, "a way" and our English *fare*.

The *philosopher* and the *sophist*, if we trust the accepted etymologies of these words, are both men of "taste." For the Greek σοφία, "wisdom," σοφός, "skilful, intelligent, wise," are close kin of σάφης, "clear, sure, unerring," a term applied to seers, etc. As the Greek word ὀπός ("juice, sap produced by incision"), the equivalent of the Latin *sapor*, proves, both σοφός and σάφης must have meant originally "of a keen, decided taste," "tasty," and then, secondarily, "clear, evident, sure unerring, wise." Colloquial American English, with its *sappy* and *saphead*, seems to have gone in the opposite direction to ancient Greek.

We have in English a famous children's rhyme, "Open your mouth, and shut your eyes, And I'll give you something to make you wise." Here taste and wisdom are correlated. The etymology of *wisdom* and *wise* does not, however, bear out the inference of the rhyme. English *wise* and its cognates in the various Teutonic languages (German *weise*, etc.) are verbal adjectives from the Indo-European radical *wid* or *wit*, "to know," which has likewise produced the English "to *wit*" and the German *wissen*, besides the Latin *videre*, "to see," which latter reveals the original meaning of the root—"to *know*" is "to *see*." The noun *wit*, which survives in a special sense in English (compare German *Witz*, etc.), is, therefore, etymologically at least, "sight," and "first sight," perhaps, not the "second sight" of the folk. From the same root come ultimately *witch* and *wiseacre*, the

last borrowed from Dutch, and replacing the Anglo-Saxon *witiga*, "prophet, soothsayer, wizard, seer," of which *witch* is a corruption. *Wizard*, too, belongs in the same category. In Modern High German the adjective *weis* also occurs, in the phrase "einem etwas *weis* machen," meaning "to hoax anybody." The extension of meaning of *wit* still survives in "to have one's *wits* about him," and the plural "wits" continues to be used as a synonym for "senses." Tennyson says:

> "Alone and warming his five wits,
> The white owl in the belfry sits."

When we cannot "see our way clear" to do anything, we are said to be "at our wits' end." The English *seer*, "prophet, wise man," is a nobler term than the similarly originating *clairvoyant* and cognates of these later days. The colloquial American *see!* belongs in this category also. German *naseweis*, "pert, saucy, inquisitive" (literally, "*nose*-wise") is also of interest here. But German has likewise *Wissenschaft*, "science." Primitive languages offer similar facts. In the speech of the Tacana Indians of Bolivia, according to Dr. D. G. Brinton (*Stud. in S. Amer. Lang.*, 1892, p. 13), the terms *babe-quisa*, "to teach," *babe-tsua*, "to learn," *babe*, "to know," are all derivatives from the root *ba*, "to see." From *ba*, "to see," comes also the term *diaba* (*dia* = "to eat"), "to taste," literally, "to eat, seeing."

Such terms as *sharp* (whence "sharper"), *acute*, and their cognates in all languages, are self-explanatory, so common is it to consider the mind *keen*, *scharfsinnig*, etc. English *shrewd*, to which commercialism has given a somewhat higher meaning than it formerly possessed ("malicious, wicked, *shrew*-like," then "cunning, acute," etc.) goes back to the animal, whose Latin name *mus aranea* betrays its evil character in folk-thought. *Astute* is a derivative from the Latin *astus*, "craft, cunning," the etymology of which is uncertain,—perhaps "city bred." A very interesting knowledge-word is English *craft* ("skill, ability, trade") which in Middle English signified "might, power, ability, art, craft, deceit,"—and the corresponding M. H. G. adjective *crafti*, "skilful, sly," our *crafty*, "cunning, sly." The cognate Modern High German word *Kraft* signifies, like Dutch *Kracht* and Danish *kraft*, "power, strength, force," but the M. H. G. and O. H. G. word had the meanings "strength, power, force of an army, multitude, abundance" and the Anglo-Saxon *craeft*, besides these, meant also "mental capacity, art, science." The final etymology of *craft* is doubtful, but Skeat suggests kinship with *cramp*, and derivation from the Teutonic radical *krap*, "to draw forcibly together." If this etymology be true, it is curious that the verb "to *cram*" is cognate. *Craft* is evidently a word which has undergone, as Kluge (*Etym. Dict.*, p. 190) points out, specialisation within the mental sphere, the English *crafty* being the latest development.

Equally interesting, perhaps, is the old Latin word *sollers*, "intelligent, skilled, sagacious, cunning, wily, crafty," derived from *sollus*, "all, entire," and

ars, "art, faculty, skill, ability, science, profession," etc.; a term having much the same significance as *craft*. The literal signification of the Latin *ars* (compare *artus*, "limb, joint") is "skill in joining, putting together,"—we still say in English "to put two and two together." The "*art* of knowledge" is "putting things together."

Somewhat similar is the etymology of the Greek τέχνη, "art, skill, craft, cunning, sleight, trade," the primitive signification of which seems to have been "a way of making or doing something,"—the word for "carpenter," τέκτων, is from the same root,—ultimately the radical *t—k*, "to shape, form, produce, generate."

The Modern High German *List* (cp. Gothic *lists*) signifies "craft, cunning, deceit," but in M. H. G. and O. H. G. it had the meanings "wisdom, prudence, slyness, cunning, art," etc.,—the cognate Middle English *liste* signified "craft, astuteness," Anglo-Saxon *list*, "art, propriety, cunning," Icelandic *list*, "prudence, skill in an art, propriety." As Kluge (p. 218) observes, the significance of the word fluctuates between "prudence" and "cunning," and the preterite—present *lais*—"I know," in Gothic, indicates the root which is also found in the German *lehren*, "to teach," *lernen* (cp. English *learn*). "to learn," etc. Cognate also are Gothic *—leis*, "knowing,", *—leisei*, "knowledge," *laisjan*, "to teach," *laiseins*, "doctrine."

To the same stock belong also English *lore* (A. S., *lar*), German *Lehre*, and their cognates in other kindred tongues. The radical at the base of all these words is *lis* with the signification "to go through," "experience,"—compare the Latin *peritia*, "experience, skill," and *experientia*, "knowledge, experience." The most primitive sense of *lis* is probably "to go," as the cognate German *Leiste*, which is the same as the English *last* (shoemaker's), Gothic *laists*, "track, goal," Anglo-Saxon *lâst* or *læst*, "footprint, track, forma," serve to indicate. The German verb *leisten*, "to perform, accomplish," O. H. G. and M. H. G., "to adhere to and execute an order, fulfil one's promise or duty," Anglo-Saxon *læstan*, "to perform, accomplish, hold, sustain, endure," belong here, as the Gothic *laistjan* (from *laists*), "to pursue, yield," proves. German *G(e)leise*, "rut, wheel-track," M. H. G. *leis* or *leise*, "trace, track," O. H. G. *leisa*, "track (of waggon)," are of the same stock, as is probably also Latin *lira*, "furrow," when *delirare*, "to be delirious, crazy, deranged, out of one's wit," literally, "to go out of the furrow." Of *lis*, the radical of all these knowledge-terms, Kluge says (p. 213): "This root has a constant tendency to pass from the sensuous meaning 'to go, follow,' into an intellectual notion.'

Another Latin word for "craft, cunning, artifice, wile" is *dolus*, probably from Greek δόλος, with like meaning, which appears also in a great many compounds expressive of "subtlety, craftiness, wiliness, cunning, treachery, deceit." The radical of these words is seen in Greek δέλεαρ, "a bait," the original idea of the root *d—l* being "to catch with a bait (as fish)." So, the wily knowledge expressed in *dolus* comes ultimately from fishing or trapping.

A common Modern High German term for "wise, knowing, prudent, shrewd," is *klug*, the etymology of which is, apparently, unknown. In M. H. G. the word signified "fine, pretty, tender, superb, brave, polite, prudent, sly." In the Low German dialects (cf. Dutch *klock*,—the word does not occur in modern English) it has the meanings "prudent, brave, great, corpulent," and in Scandinavian *klôkr* = "prudent, cunning." Probably the Middle English *glen*, "wise," Anglo-Saxon *gllaw*, "sagacious," is the same word; perhaps also the Scotch *gleg*, "quick of eye," belongs here. The last may reveal the original signification.

There is a very interesting group of knowledge-words, represented by the English loan-words from Latin *apprehend*, *comprehend*, *perceive*, *conceive*, etc., whose very physical basis is revealed in the common expression, "I don't quite *grasp* (or *get*) your meaning." In Latin *apprehendere* signifies "to catch, lay hold of, take, seize, embrace," *comprehendere*, "to lay hold of, catch, apprehend, comprehend, comprise, embrace, perceive, observe, see, understand," etc., and the simpler *prehendere*, "to grasp, seize, catch, take, take in with the eye, seize, apprehend, comprehend." *Prehendere* (the Greek cognate is χανδάειν, "to take in, hold, be able") is derived from the prefix *pre*, "before," *hendere*, "to seize, get," a word actually akin to the English *get*. So, when we *seize* a point in an argument, *grasp* the situation, or, in the colloquialism of the day, *get on* to anything, we are repeating the phenomena revealed to us by these etymologies. Indeed, what real difference is there between the Latin *apprehendisne* and the street Arab's "*D'ye catch on?*" And in quarters where the language of the street and the forum is tabooed we often hear it said: "I don't *catch* your meaning."

The chief component of *percipere* and *concipere*, which have in Latin both the physical and the psychical significations," is *capere*, "to take hold of, seize, grasp." So, the psychological terms *concept*, *percept*, *recept*, *apperception*, etc., go back to a physical basis. The German verb, *begreifen*, "to comprehend, understand, conceive," and the substantive *Begriff* "conception, idea, notion," are, in like manner derived from the radical represented by *greifen*, "to seize, grasp. lay hold, grip." In English we say "he has lost his *grip*," even with reference to things intellectual.

Like *klug* in German, *clever*, which in English signifies "skilful, dexterous," and as an Americanism "good-natured, with wits below smart,"—in English dialect with meanings including all of these, including "bright (intellectually),"—is of doubtful etymology; Skeat, with apparent justice, considers *clever* to be a metamorphosis of Middle English, "quick, active, nimble, agile," from Old French *delivre*, "quick," literally "freed" (from Latin *de*, *liber*). The oldest meaning of *clever* would seem to be "ready (able) to seize," which brings it into the category of *grasp*-words.

The English *understanding*, *to understand* and the Modern High German *Verstand* ("understanding, intelligence, sense, reason") *verstehen* ("to understand") are very interesting knowledge-terms, whose etymology has not been alto-

gether satisfactorily explained. In M. H. G. *verstant*, a rare word, signifies "explanation," while M. H. G. *verstandnisse*, O. H. G. *firstantnissi*, has the meanings, "intelligence, insight, understanding,"—the verb, M. H. G. *verstân*, O. H. G. *firstân*, signifies "to perceive, see into, notice, understand." Of these words Kluge remarks (p. 377): "How the meaning can be derived from the root of *stehen* is not clear; it is usually related to Greek ἐπίσταμαι, 'understand,' compared with the root *sta*, 'to stand,'" English *understand* presents the same problem. Skeat, however, interprets the Anglo-Saxon *understandan* (from *under*, *standan*) as signifying, literally, "to stand under, or among," and hence "to comprehend," comparing for signification and etymology the Latin *intellegere*, from which comes *intellectus* (whence English *intellect*) *intelligencia* (whence English *intelligence*) and their derivatives in many languages of Europe. *Intelligere* signifies, literally, "to choose between" (*inter*, *legere*), the chief component, *legere*, having the meaning "to gather, pick up," probably as a bird does grains, seeds, etc.

Epistemology, the term by which the philosophers designate the "theory of knowledge," has as its first component the Greek ἐπιστήμη, "knowledge, understanding, skill, experience, wisdom, science," a derivative from the verb ἐπίσταμαι, "to understand, be capable, know how," which is itself derived from the particle ἐπί and ἵστημι, "to cause to stand, erect, establish, place, make fast, fix." The word ἵστημι is lengthened from στάω, from the same Indo-European radical as English *stand*, German *stehen*, and Latin *stare*. There is thus abundant evidence to show that "where one *stands*," with reference to any matter, is a very ancient definition of his knowledge. From the Zapotec Indian to the modern epistemologist is a long way, but language has much of interest wherewith to beguile us as we go.

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CRITICISMS AND DISCUSSIONS.

HILBERT'S FOUNDATIONS OF GEOMETRY.[1]

Since its appearance in 1899 Hilbert's work on *The Foundations of Geometry* has had a wider circulation than any other modern essay in the realms of pure mathematics. As types of the appreciation which it has received, we may refer to the reviews of Sommer,[2] Poincaré,[3] Halsted,[4] and Hedrick.[5] In each case Hilbert's achievement is enthusiastically praised, as well as (in the latter two cases) the enterprise of the publishers and translator of the English edition. Dr. Hedrick's review in particular is valuable for correcting certain errors of detail in the translation that are of importance to the mathematical reader. Since the purpose of the present article is to point out the relation of Hilbert's work to certain less widely known investigations of the same subject, we can here only add the opinion that the translation is a very competent and useful piece of work.

Study of the foundations of geometry began with no more recent a mathematician than Euclid. In giving form to the hitherto unorganised mass of geometrical knowledge, it was necessary to ask the question: What are the statements that it is necessary to presuppose in order logically to deduce from them the properties of space? Euclid's reply to this conundrum was supposed for many centuries to be the only one possible. The non-Euclidean geometry which arose out of vain attempts to prove the axiom of parallels a consequence of the other axioms was the first break in this tradition. Its development has been brilliant and interesting, but we must summarise it in a list of names: Saccheri (1733), Legendre, Gauss, Schweikart, Lobatschewsky, Bolyai, Riemann, Beltrami, Helmholtz, Cayley, Klein, Lie.

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[1] *The Foundations of Geometry*, D. Hilbert. Authorised translation by E. J. Townsend, Ph. D. Chicago: The Open Court Publishing Company, 1902. 8vo. Pages, vii, 132.

[2] An English translation of Sommer's article appeared in the *Bulletin of the American Mathematical Society*, Vol. VI., p. 287. 1900.

[3] *Bulletin des Sciences Mathématiques*, Vol. XXVI., p. 249, September, 1902.

[4] *The Open Court*, September, 1902.

[5] *Bulletin of the American Mathematical Society*, Vol. IX. (1902), p. 158.

The non-Euclidean geometers, however, confined their attention in the main to the axiom of parallel lines, and it was not till about twenty years ago that an attempt was made to recast the entire system. In 1882, in a book called *Vorlesungen über die neuere Geometrie*, Dr. Moritz Pasch published a system of axioms and showed how it was possible from them to deduce the theorems of geometry. The work of Pasch was considerably simplified and improved by Peano,[1] who attacked the problem with the aid of a ratiocinative calculus. An independent attempt was made by another Italian, Veronese,[2] who studied in particular the continuity of space. Veronese's book, which is somewhat obscure, has received a helpful commentary from T. Levi-Civita.[3]

At the time of Hilbert's investigations, the term "mathematical science" had acquired a precise technical meaning. A mathematical science consists of a body of propositions stated about certain elements in terms of certain relations. The elements in Hilbert's system, for example, are denoted by the words "point," "line," "plane." The relations are indicated by such words as "are situated," "between," "congruent." An element is understood to be quite independent of any mental image that may accompany it in our thinking. It has no properties except such as are stated in the propositions. The axioms are a system of propositions such that:

1. Every proposition of the mathematical science in question can be deduced from the axioms.
2. No axiom can be deduced from the other axioms.

Hilbert arranges his axioms in five groups according to the relations to which they give meaning.

I, 1–7. Axioms of connection (involving the term "are situated").
II, 1–5. Axioms of order (involving the term "between").
III, Axiom of parallels.
IV, 1–6. Axioms of congruence.
V, Axiom of continuity.

There are thus twenty axioms in Hilbert's system.

The sense in which an element is independent of the mental images accompanying it is made clear by the method of proving an axiom to be independent,—i. e., not deducible from the others. This is done simply by exhibiting a self-consistent mathematical science, in which all the axioms except the one in question are valid propositions, whereas that one is not. For if the indicted axiom were deducible from the others, it would be a true proposition of the new science.

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[1] Peano, *I principii de Geometria*, Turin, 1889; "Sui Fondamenti della Geometria," *Rivista di Matematica*, Vol. 4 (1894), pp. 51–90.

[2] Veronese, *Fondamenti di Geometria a più dimensioni e a più specie de unità rettilinee*. Padua, 1891. German translation by Schepp, Leipzig, 1894.

[3] *Memorie della Reale Accademia dei Lincei Roma*, Vol. 7, pp. 91–96, 113–121; 1898.

Take or example the classical case of the Euclidean axiom, Hilbert's III. A mathematical science of the kind required is furnished by the propositions about the points interior to an ordinary sphere. Let the significance in our new science of the words

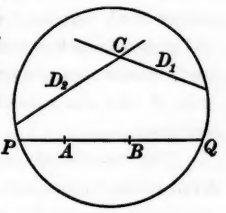
"point" be interior point of the sphere S (not including the boundary of sphere),

"line" be portion of line interior to the sphere S,

"plane" be portion of plane interior to the sphere S.

The relations of order and connection of "points," "lines," and "planes" are as usual, but on a "line" l (which is the portion of the ordinary line bounded by the points P and Q) the "distance" apart of two points A and B shall be

$$\log\left(\frac{AP.BQ}{AQ.BP}\right)$$



where AP, etc., are distances in the usual sense.

With a similar convention about the meaning of the word "angle," a very little mathematics suffices to show that all the axioms of the groups I, II, IV, V (and of course the theorems deducible from them) are valid propositions about our new "points," "lines," etc. For example, "space" is infinite, for if A is moved toward Q, the "distance AB," as defined, increases indefinitely.

Moreover, unless the Euclidean geometry is at war with itself, these propositions contain no self-contradiction; for if they did, on translating them back into the corresponding properties of the sphere the contradiction would remain and therefore the Euclidean geometry would contain self-contradictions. Now if the parallel axiom were deducible from I, II, IV, V, we should be able to obtain the proposition that through a point C there is one and *only* one "line" which does not meet the "line" AB. It is plain at once that there are *many* "lines" (CD_1, CD_2, for example) that do not meet AB. The proof is therefore complete that III cannot be deduced from I, II, IV, V.

It was generally supposed[1] till the researches of Peano and Hilbert became known, that even though this kind of analysis could be applied to the parallel axiom, there might be other axioms to which it would not apply. It has by this time, however, been extended to all the axioms both of arithmetic and geometry. The mental images that are applied to the words "point" and "line," etc., in the bizarre mathematical sciences that have been used for this purpose vary from the letters of the alphabet to more or less complicated mathematical curves. The possibility is also recognised of using different mental images in the same science. For example, a distortion can be conceived to be applied to space in such a way that

[1] Cf. Poincaré, *Nature*, Feb. 25, 1892; Russell, *The Foundations of Geometry*, Introduction, p. 6, 1897, Cambridge, University Press.

all the lines are bent without altering their mutual relations. It is evident that if we think the geometrical relations in terms of the bent lines our images are changed without affecting the form of the axioms or the method of deduction. Theoretically, at least, the deductions could be made without any reference to their content by the use of a ratiocinative calculus like that of Peano or a Jevons logical machine.

The essential point is the recognition of the processes of logic, as separate from the mental imagery of the "elements." For this reason it is perhaps well to use the word "symbol," instead of "element" and "relation" as that, in the formal process, which answers to a basal concept of a science. The symbols of Hilbert's science, accordingly, are "points," "lines," "between," etc. The symbolic point of view also makes it possible to state in simple language what is meant by a definition.

Theoretically, all the propositions of a mathematical science can be stated in terms of the fundamental "elements" and their relations. As a rough example—we should never need to speak of "circles" but only of "sets of points in a plane equally distant from a given point." It is however convenient to introduce the term "circle" by a definition. Again, in arithmetic we may have the *definition*, $3+1=4$. In general, we mean by a definition an agreement that permits us to substitute a simple term or symbol for more complex terms or symbols. The only symbols that are undefined are the basal elements.

The question immediately arises, is it perhaps possible to define some of the elements in terms of the other elements? In that case our system of assumptions is clearly not as simple as possible, even if no one of the axioms is deducible from the others. We have therefore, in addition to the problem of determining a set of independent axioms, the analogous problem of determining an irreducible set of elements, i. e., a system of elements such that no one of them can be defined in terms of the others.[1]

The two problems are evidently interrelated, and Hilbert's geometry furnished a good example of that fact. Axiom I, 3 reads as follows:

I, 3: Three points A, B, C, not situated in the same straight line, always completely determine a plane α. We write $ABC=\alpha$.

This axiom was stated by Prof. F. Schur[2] of Karlsruhe to be deducible from the rest, but Professor E. H. Moore of Chicago, in the paper cited below, proved that such a statement is incorrect under the literal interpretation of Hilbert whereby a

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[1] This problem has been attacked and stated in a general, though somewhat different, form by Professor A. Padoa of the University of Rome. See *Bibliothèque du Congrès International de Philosophie, III. Logique et Histoire des Sciences.* Page 309, 1901. The method he proposes seems hardly adequate.

[2] Schur, "Ueber die Grundlagen der Geometrie," *Mathematische Annalen*, Vol. LV. (1901), pp. 265–292.

plane is made one of the undefined elements. The symbol "plane" is capable of definition in terms of "point" and "line," but unless this definition is admitted the axiom I, 3 is not a consequence of the other axioms. We thus have at hand a case showing how a system of axioms may be non-redundant while the system of elements is not irreducible.

It is along this direction that the severest criticism of Hilbert's work can be made. While his axioms are in the main independent, his system of elements is far from irreducible. In the axioms of the groups I and II there are three elements, points, lines, and planes, and two relations, represented by the phrases "are situated" and "between." It may be here stated, leaving the proof for a more technical article, that all these elements and relations can be defined in terms of "points" and "betweenness."[1]

Hilbert himself defines the relations of groups III and V in terms of the other relations. But he does not make use of the fact pointed out some twenty years ago by Klein,[2] that the congruence relation of group IV (which amounts to the same thing as rigid motion) can also be defined.

With respect to the congruence axioms, Hilbert makes[3] a curiously misleading statement:

> "We shall show the independence of the axioms of congruence by demonstrating that axiom IV, 6 . . . . cannot be deduced from the remaining axioms I, II, III, IV, 1–5, V, by any logical process of reasoning."

If we agree that congruence is an *element* it is obvious without further proof that the axioms of IV cannot be deduced from I, II, III, V, because congruence is not mentioned in I, II, III, V. What Hilbert really does, however, is to exhibit a mathematical science in which I, II, III, V are satisfied, and in which there is given a definition of congruence that verifies IV, 1–5, without verifying IV, 6. The effect of this is merely to demonstrate that a certain relation has the first five without the sixth property of congruence. The possibility still remains that by means of a suitable definition all the propositions of IV can be logically deduced.

The independence of his axioms was not fully investigated by Hilbert himself, but further research has revealed only one axiom which is capable of being deduced from the other axioms if the elements are retained precisely as Hilbert gave

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[1] For other ways of reducing to two the number of "elements" see Padoa "Un nouveau systèm de définitions pour la géométrie euclidienne." *2d. congrès international des mathématiciens*, 1900, p. 353. Reference is there made by Padoa to related work of Pieri.

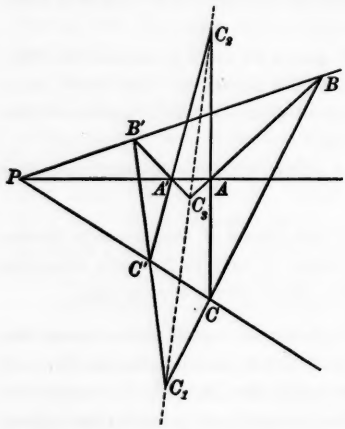
[2] Klein, *Math. Ann.*, Vols. IV., p. 573, and VI., p. 271 (1871–1873). It is important to add that the definition of congruence is not unique in the same sense as is the definition of "plane." In other words, by the Klein definition, rigid motion is a special case of a more general kind of "transformation of space."

[3] Art. 11, p. 32 of the translation.

them. This is axiom II, 4, the discovery of whose redundance was made a little over a year ago by Professor Moore.[1]

It is important to remark after the criticisms just cited that the imperfections of his axioms need not impair the importance of Hilbert's contribution to geometry. His work on axioms, indeed, can hardly be called more original or elegant than that of the Peano school in Italy. He has, however, by an attractive form of presentation drawn the attention of the entire mathematical world to one of the most fertile fields of investigation both for the technical mathematician and for the philosopher.

Besides this, Hilbert has extended the scope of his investigations so as to apply to any theorem whatever and not merely to the axioms. Take for example the famous theorem of Desargues:



> If two triangles are so situated in a plane that the lines joining their corresponding vertices AA', BB', CC', meet in a point P and the corresponding sides AB, $A'B'$, etc., meet in three points, C_1, C_2, C_3; then C_1, C_2, C_3 are collinear.

This theorem evidently has not to do with the metrical properties of the triangle. It becomes obviously true if the two triangles are not in the same plane, for then the corresponding sides can meet only in the line of intersection of the two planes. Nevertheless the theorem, when the two triangles are in the same plane, can *not* be demonstrated in a non-metrical way unless a construction is used which involves points outside the plane of the two triangles. This fact had long been suspected but never, before Hilbert's time, proved. The method of proof is an extension of that used to prove the independence of an axiom. A mathematical science is exhibited in which all the non-metrical axioms about the plane are valid, but in which it is not assumed that there are points outside the plane. In this mathematical science[2] the Desargues

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[1] E. H. Moore, "On the Projective Axioms of Geometry," *Transactions of the American Mathematical Society*, Vol. III., p. 142, January, 1902. A somewhat simpler proof, due to Mr. R. L. Moore, was published in an article on "The Betweenness Assumptions," *American Mathematical Monthly*, April, 1902.

[2] A simpler example of a non-Desarguean science than that of Hilbert is given by Dr. F. R. Moulton, *Transactions of the American Mathematical Society*, Vol. III. (1902), p. 192.

proposition is not a valid theorem, and therefore cannot be deduced from the axioms mentioned.

The extension of this method to the Desargues theorem and to many others is. I believe, the main achievement of Hilbert and of his students. In a sense it is the final extension to geometry of a tendency that has been powerful in the realms of analysis for the last half century. This is the desire, on the one hand, for perfectly rigorous logic in demonstration, and on the other hand to state theorems in the form of necessary and sufficient conditions.

A necessary and sufficient statement is of this form: "If A, then B, and if B, then A." Both the direct statement and the converse of a theorem are to hold. Stated in this language, the problem of the foundations of geometry is to find a system of axioms that is necessary and sufficient for geometry,—necessary, in that no axiom can be dispensed with, and sufficient for the deduction of the whole system of geometrical knowledge. In like manner the basal elements are to be necessary and sufficient for the definition of every other symbol of the science. The investigation of each theorem ought to be such that we can say that 'axioms a, b, c are necessary and sufficient for the validity of theorem D.' Finally, of each definition we should like to say that it is sufficient to determine the thing defined, whereas each symbol entering into it is necessary for such determination. The formulation of a science with this degree of necessity and sufficiency is the ideal of the investigators of the foundations of geometry.

OSWALD VEBLEN.

THE UNIVERSITY OF CHICAGO, December 1, 1902.

BOOK REVIEWS.

BABEL AND BIBLE. A Lecture on the Significance of Assyriological Research for Religion. Delivered before the German Emperor by *Dr. Friedrich Delitzsch*, Professor of Assyriology in the University of Berlin. Translated from the German by Thomas J. McCormack. Profusely illustrated. Chicago: The Open Court Publishing Company. London: Kegan Paul. Trench, Trübner & Co., Ltd. 1902. Pages, 66. Price, 50 cents net (2s. 6d. net).

The German Oriental Society has been making extraordinary and praiseworthy efforts of late to direct the attention of their government to the cultural importance of the excavations now being conducted in Mesopotamia and to induce it to grant generous subsidies for the further prosecution of this work by German scholars. The activity of Germany in these explorations has not been so great as that of England or France, and the measures referred to have been taken in order that the Fatherland might not lag behind in the international competition in practical Assyriology. Lectures were accordingly instituted for arousing popular as well as governmental interest in the work of Babylonian exploration, and were afterwards published in a series entitled *Der Alte Orient*; their authors are such men as Delitzsch, Winckler, Wiedemann, Zimmern, etc.

Of all these lectures that which excited the most widespread attention was one on "Babel and Bible" which Dr. Friedrich Delitzsch, one of the greatest leaders of Assyriological research, delivered in January, 1901, before the Emperor of Germany, at a regular meeting of the Oriental Society, and which he was specially invited to deliver a second time before the Emperor in the Royal Castle at Berlin. The lecture was subsequently published with profuse illustrations by Hinrichs of Leipsic, and it now appears in an English translation. The American publishers have increased the number of illustrations of the German edition, and have had them greatly enlarged. The lecture is a graphic and glowing popular *résumé* of the influence which Assyriological research has exerted on the interpretation of the Bible, and presents in succinct form a general view of Assyrian and Babylonian history, art, literature, and civilisation. There is now no excuse for any one's remaining unacquainted with the significant and astounding work which is being

done in the Orient, or for remaining blind to the light which this work has thrown on the entire field of early history. It is a fascinating subject, and Dr. Delitzsch's lecture has portrayed it at its best.

DIE BABYLONISCHE KULTUR IN IHREN BEZIEHUNGEN ZUR UNSRIGEN. EIN VORTRAG. Von *Hugo Winckler*. Leipzig: J. C. Hinrichs'sche Buchhandlung. 1902. Pages, 53. Price, M. 80.

Prof. Hugo Winckler of Berlin, a colleague of Prof. Friedrich Delitzsch, condenses in a short lecture of fifty-three pages the main facts known about Babylonian civilisation. He finds that Babylonian views have dominated the development of mankind in Greece and Rome during the Middle Ages, and are to a great extent preserved even to-day. This is especially true in mathematics and astronomy, in the names of the months, of the zodiac, and in popular expressions. Even the festivals of the ancient Babylonians are still continued among us under other names; and finally, our folklore can be rediscovered in the ancient legends and religious notions of those nations who dwelt thousands of years before us in Mesopotamia. P. C.

DIE GESETZE HAMMURABIS, KÖNIGS VON BABYLON, UM 2250 V. CHR. Das älteste Gesetzbuch der Welt. Uebersetzt von *Dr. Hugo Winckler*. Mit einer Abbildung des Steindenkmals. Leipzig: J. C. Hinrichs'sche Buchhandlung. 1902. Pages, 42.

This pamphlet, published by the *Vorderasiatische Gesellschaft*, is a German translation of an ancient stone slab containing the laws of Hammurabi, which is one of the most interesting documents of old Babylonian history ever recovered in the Orient. Hammurabi is the Biblical Amraphel mentioned in Genesis xiv. as a contemporary of Abraham and King of Shinar. He was one of the greatest kings of Babylon, a reformer and law-giver. After a period of great unrest, he restored order and re-established a stable condition of society. His laws form a landmark, not only in the history of Babylon, but in the history of mankind. They are more than 700 years older than the laws of Moses, according to the orthodox tradition, supposing the latter to be given in 1500 B. C.

Hammurabi flourished about 2250 B. C. The remarkable stone on which the inscription is recorded represents King Hammurabi in the presence of his god (the sun-god of Sippar), who communicates to him the laws which are codified in the mooted inscription deciphered for the first time by Father V. Scheil.

It was customary in ancient times for conquerors to transfer statues of gods and other significant monuments to their own capital and to replace the original inscription by a proclamation of their own triumphs. Our stone suffered a similar fate, but by good fortune the text of the inscription was saved. Five lines only are erased, and the victor's name does not appear on it. Moreover, even these five

missing lines have been restored from a copy of the laws of Hammurabi that was recovered from the library of Assurbanipal.

The contents of Hammurabi's laws are remarkable for their fairness, their humane principles, and their detailed accuracy. The inscription begins with a personal enunciation of King Hammurabi; he proclaims that he owes his government and success to the grace of the gods, and promises to rise over mankind as Shamash the sun-god does over the earth. His purpose is to protect the weak ones, to destroy evil doers, and to procure justice to every one. The law book contains 282 single statements, and deals with the questions of practical life,—unjust accusations, marriage relations, contracts, rent, the regulation of inheritances, regulations of the market, and similar ordinances. P. C.

THE CREATION STORY OF GENESIS I. A Sumerian Theogony and Cosmogony. By *Dr. Hugo Radau*. Chicago: The Open Court Publishing Company. London: Kegan Paul, Trench, Trübner & Co., Ltd. 1902. Pages, vi, 70. Price, Boards, 75 cents net (3s. 6d. net).

The greater part of this booklet appeared in *The Monist* for July, 1902. It considers the creation story of the first chapter of Genesis in the light of the oldest sources at our disposal, viz., the Sumerian cosmogony and theogony, of which the Biblical creation story of the Priestly School is a redaction. For having criticised and eliminated the mythical elements of the Sumerian tradition, P. is called by Dr. Radau "the first Higher Critic." Dr. Radau, in explaining certain attributes of the gods, was compelled to touch briefly upon the Sumerian cosmology, of which he has given a new representation. He also makes reference to the Arabian Pantheon. The little book is apt to appeal only to specialists and Old Testament and comparative Semitic research.

HYMNS OF THE FAITH (DHAMMAPADA). Being an Ancient Anthology Preserved in the Short Collection of the Sacred Scriptures of the Buddhists. Translated from the Pâli by *Albert J. Edmunds*. Chicago: The Open Court Publishing Company. London: Kegan Paul, Trench, Trübner & Co., Ltd. 1902. Pages, xiii, 110. Price, $1.00 (4s. 6d. net).

This celebrated and ancient anthology of Buddhist devotional poetry was compiled from three sources: (1) The utterances of the Buddha and his disciples; (2) From early hymns by monks; and (3) From popular poetic proverbs of India. It is the Buddhist *Imitation of Christ*, and constitutes one of the noblest of the world's religious expressions. Its fame relieves us from commenting upon the nature of its contents; the eloquent words of the translator, Mr. Edmunds, best characterise it. He says:

"If ever an immortal classic was produced upon the continent of Asia, it is this. Its sonorous rolls of rhythm are nothing short of inspired. . . . No trite ephemeral songs are here, but red-hot lava from the abysses of the human soul, in one out of the two of its most historic eruptions. These old refrains from a life

beyond time and sense, as it was wrought out by generations of earnest thinkers, have been fire to many a muse. They burned in the brains of the Chinese pilgrims, who braved the blasts of the Mongolian desert, climbed the cliffs of the Himâlayas, swung by the rope-bridge across the Indus where it rages through its gloomiest gorge, and faced the bandit and the beast, to peregrinate the Holy Land of their religion, and tread in the footsteps of the Master. Verses were graven on the walls of august temples at the command of Hindû emperors who abolished capital punishment, mitigated slavery, and established hospitals for men and animals, under the sway of this marvellous cult; and by Ceylon monarchs whose ruined reservoirs, as large as lakes, astonish us among the wonders of antiquity. And to-day, after twenty centuries of Roman and Christian culture, they have won the admiration of Europeans and Americans in every seat of learning, from Copenhagen to the Cambridges, and from Chicago to St. Petersburgh."

Mr. Edmunds has given us a very literal translation, but he has also had a literary ideal in view and sought by using an archaic and poetic style "to convey some flavor of the original" to his readers. While he remarks that it is "too ambitious a wish" to hope to naturalise in English this Buddhist holy writ as the King James Version has naturalised the Christian, an unbiassed critic can only say that he has contributed much toward this end. In this little book a living knowledge of the main Buddhistic doctrines is placed within the reach of every reader.

SUR LES PRINCIPES FONDAMENTAUX DE LA THÉORIE DES NOMBRES ET DE LA GÉOMÉTRIE. Par *H. Laurent*. Paris: C. Naud, Publisher. Pages, 68 Price, 2 francs.

In this number (No. 20) of the admirable scientific series called *Scientia*, devoted to the discussion of the burning questions of contemporaneous research, the distinguished French mathematician M. Laurent has considered the bases of the theory of numbers and of the theory of space. M. Laurent rejects axioms and begins with hypotheses. "An axiom," he says, "is a proposition which atavism, education, and the authority of our teachers have obliged us to accept as true without critical examination." His hypotheses are: "(1) I reason correctly (the hypothesis of all reasoning); (2) Quantity is to be defined (I suppose that quantity exists); (3) There are quantities susceptible of being divided into as many equal parts as one wishes (I shall define division into equal parts); (4) When a quantity increases or diminishes continuously without becoming larger or smaller than a fixed quantity it has a limit." "A truth is a plausible hypothesis or a logical consequence of non-contradictory hypotheses."

On this small number of hypotheses he seeks to found the theory of numbers. His point of view is utilitarian as distinguished from the æsthetic point of view of Tannery, Méray, and Cosserat. His conclusions he thinks to be just as simple as, and certainly more accessible to young minds than theirs. He is averse to their disinclination to base the theory of numbers on the consideration of quantity, and

he remarks that since mathematics is a science having for its aim the study of the exact and necessary relations of magnitude, of the form and position of the objects accessible to our senses, therefore the above-named gentlemen are pursuing not mathematical but metaphysical researches. For them pure mathematics is an exercise in logic; for M. Laurent its object is the study of quantities with the help of numbers. Thirty pages of the work are devoted to the consideration of numbers and thirty-eight pages to that of pan-geometry. He concludes that the plane is an arbitrary surface, or rather that geometry could be taught without defining a plane but by merely calling it an arbitrary surface of which it is impossible to give a precise definition. Geometry is a physical and experimental science,—a language competent to co-ordinate facts, or rather *appearances*, and indifferently Euclidean or non-Euclidean. The Euclidean geometry is that which appeared most simple to the brain of our ancestors. It would not be astonishing if there existed intelligent beings on the planet Mars with a non-Euclidean geometry and with different ideas from ours regarding displacement without change of form.

ESSAYS ON THE THEORY OF NUMBERS. I. Continuity and Irrational Numbers. II. The Nature and Meaning of Numbers. By *Richard Dedekind*. Authorised Translation by Wooster Woodruff Beman, Professor of Mathematics in the University of Michigan. Chicago: The Open Court Publishing Company. London: Kegan Paul, Trench, Trübner & Co., Ltd. 1901. Pages, 115. Price, 75 cents.

The mathematical reading public unacquainted with German is under considerable obligation to Professor Beman for the present faithful rendering of these two celebrated essays of Dedekind. Modern logical views of continuity and arithmetic are largely based on the results which Dedekind and his contemporary, G. Cantor, furnished (the first of their essays was published in 1872), and it is good that these investigations should be made accessible to all readers in their original form. Furthermore, the German of these essays is not the easiest imaginable reading, and the interpretation of the forms of expression which Professor Beman has given and which has involved considerable study, will also be welcome to readers of German who are in the habit of purchasing but not perusing German books.

"In science," remarks Dedekind, "nothing capable of proof ought to be accepted without proof." But in laying the foundations of the simplest of the sciences, "viz., that part of logic which deals with the theory of numbers," this postulate has not been complied with. In saying that arithmetic (algebra and analysis) is a branch of logic, Dedekind by implication asserts that the number-concept "is entirely independent of the notions or intuitions of space and time . . . an immediate result of the laws of thought." His answer to the question of the nature and meaning of numbers is that "numbers are free creations of the human mind and serve as a means of apprehending more easily and more sharply the difference of things."

He continues in the preface to his essay of 1887: "It is only through the purely logical process of building up the science of numbers and by thus acquiring the continuous number-domain that we are prepared accurately to investigate our notions of space and time by bringing them into relation with this number-domain created in our mind. If we scrutinise closely what is done in counting an aggregate or number of things, we are led to consider the ability of the mind to relate things to things, to let a thing correspond to a thing, or to represent a thing by a thing, an ability without which no thinking is possible. Upon this unique and therefore absolutely indispensable foundation, . . . must, in my judgment, the whole science of numbers be established."

Such is the fundamental idea of Dedekind's thought. He succeeded by means of it in building up a continuous number-system which included not only rational numbers but also, and *necessarily*, irrational numbers. These memoirs can, as their own author remarks, "be understood by any one possessing what is usually called good common sense; no technical philosophic, or mathematical, knowledge is in the least degree required."

ELEMENTE DER STEREOMETRIE. Von *Prof. Dr. Gustav Holzmüller* in Hagen i. W. Vierter Teil, Fortsetzung der schwierigeren Untersuchungen. Mit 89 Figuren. Leipzig: G. J. Göschen'sche Verlagshandlung. 1902. Pages, xi, 311. Price, bound, 9.50 Marks.

HÖHERE ANALYSIS. Zweiter Teil. Integralrechnung. Mit 89 Figuren. Von *Dr. Friedrich Junker*, Professor am Realgymnasium und an der Realanstalt in Ulm. Zweite, verbesserte Auflage. Leipzig: G. J. Göschen'sche Verlagshandlung. 1901. Pages, 209. Price, 80 Pf.

PROJEKTIVE GEOMETRIE. In synthetischer Behandlung. Von *Dr. Karl Doehlemann*, Privatdozent an der Universität München. Zweite, vermehrte und verbesserte Auflage. Mit 85 Figuren. Leipzig: G. J. Göschen'sche Verlagshandlung. 1901. Pages, 176. Price, 80 Pf.

DARSTELLENDE GEOMETRIE. Erster Teil. Elemente; Ebenflächige Gebilde. Von *Dr. Robert Haussner*, Professor an der Universität Giessen. Leipzig: G. J. Göschen'sche Verlagshandlung. 1902. Pages, 192. Price, 80 Pf.

REPETITORIUM UND AUFGABENSAMMLUNG ZUR INTEGRALRECHNUNG. Von *Dr. Fr. Junker*, Professor am Realgymnasium und an der Realanstalt in Ulm. Mit 50 Figuren im Text. Leipzig: G. J. Göschen'sche Verlagshandlung. 1902. Pages, 130. Price, 80 Pf.

REPETITORIUM UND AUFGABENSAMMLUNG ZUR DIFFERENTIALRECHNUNG. Von *Dr. Fr. Junker*, Professor am Realgymnasium und an der Realanstalt in Ulm. Mit 42 Figuren im Text. Leipzig: G. J. Göschen'sche Verlagshandlung. 1902. Pages, 119. Price, 80 Pf.

The domain of pure elementary geometry was immensely extended by the researches of Poncelet, Chasles, Steiner, and Möbius, and the reaction which set in

during the nineteenth century against the purely analytical methods of the eighteenth has assumed so formidable proportions that the author of the first work above listed, Dr. Holzmüller, even intimates the *possibility* that analysis may at some future time be overtaken by geometry. He has himself been surprised at the simplicity of the methods and of the form of the results which he has reached in his purely geometrical developments without the assistance of the higher analysis, and many of his colleagues have joyfully recorded a like astonishment. It has not been the author's intention to render the calculus superfluous in the treatment of solid geometry, but merely to *systematise* the elementary methods, and to exhibit and apply them in the full extent of their new-born power, hoping in this way to obtain as many as possible new points of view for the subjects treated and to strengthen the student's powers of spatial visualisation. The present and last volume of this extensive and scholarly work on solid geometry presents the elements of the theory of certain moments of homogeneous point-systems, particularly static and centrifugal moments and moments of inertia; applies in their full generality the summation formulæ; gives stereometric and mechanical interpretations and visualisations of the above-mentioned moments for plane surfaces and curves; discusses the catenoid, Gauss's pseudo-spherical sphere; etc., etc.; the many subjects treated being too numerous to mention in detail. Elucidative historical and methodological remarks are interwoven in the text, all combining to make Holzmüller's work the most thorough and helpful existing treatise on the elements of solid geometry as developed without the use of the calculus. The publishers have done their utmost to render the work serviceable. Good indexes have been added.

The remaining five works listed at the head of this notice belong to the very cheap and admirable series of little mathematical text-books called the "Sammlung Göschen." The little treatise on the integral calculus by Dr. Junker is, as we have had occasion to learn in detail, a concise and adequate presentation of the subject. It consists of ten chapters treating respectively of the integration of simple, rational, irrational, and transcendental differentials, of definite integrals, of the application of the integral calculus to geometry of two and three dimensions and to statics, and of the double integral and its applications, giving finally a brief discussion of ordinary differential equations. Dr. Junker has also compiled (see the fifth work above listed) a syllabus of the formulæ of the integral calculus and a collection of examples for practicing it. He has done the same for the differential calculus (see the sixth work above listed), which is issued as a species of supplement of his work on the differential calculus in this same series. These little books can be used independently of their companion works, inasmuch as in each case the formulæ precede the exercises, and illustrative examples are worked out at the beginnings of each section. They are also supplied with figures. Little knowledge of German is requisite to use these books profitably.

Dr. Doehlemann's book on *Projective Geometry* is intended for beginners who are just about entering on the study of modern synthetic geometry. The subject

has been attacked from different sides, not excluding that of metrical geometry, although of course the chief emphasis has been placed on constructive and visualising methods. The geometry of space is considered in connection with that of the plane. The figures are in two colors.

Dr. Robert Haussner, the author of the *Descriptive Geometry* in this series, is professor in the University of Giessen and the translator of Monge's celebrated *Géométrie descriptive*, which was the pioneer work of this very practical and useful branch of mathematics, and the inaugurator of the modern Renaissance of geometrical research. Dr. Haussner's work, while concise, is very comprehensive and a thoroughly modern presentation of its subject.

It will not be amiss to mention in this connexion that Göschen has also recently issued the first part of a *Multidimensional Geometry* by Dr. P. H. Schoute, of the University of Groningen, Holland. The book treats of linear spaces and the work will restrict its investigations to the geometry of Euclid. *μ.*

THE SCIENCE OF MECHANICS. A Critical and Historical Account of Its Development. By *Dr. Ernst Mach*, Professor of the History and Theory of Inductive Science in the University of Vienna. Translated from the German by Thomas J. McCormack. Second Revised and Enlarged Edition. With 259 Cuts and Illustrations. Chicago: The Open Court Publishing Company. London: Kegan Paul, Trench, Trübner & Co. 1902. Pages, xix, 605. Price, Cloth, $2.00 net (9s. 6d. net).

The appearance of the second edition of Dr. Ernst Mach's now famous work on the development of mechanics is coincident with a wide-spread revival of philosophical interest in the foundations and methods of the various exact sciences; and the translator of the present work remarks:

"Since the appearance of the first edition of the present translation of Mach's *Mechanics*, the views which Professor Mach has advanced on the philosophy of science have found wide and steadily increasing acceptance. Many fruitful and elucidative controversies have sprung from his discussions of the historical, logical, and psychological foundations of physical science, and in consideration of the great ideal success which his works have latterly met with in Continental Europe, the time seems ripe for a still wider dissemination of his views in English-speaking countries. The study of the history and theory of science is finding fuller and fuller recognition in our universities, and it is to be hoped that the present exemplary treatment of the simplest and most typical branch of physics will stimulate further progress in this direction."

The text of the present edition of the English translation contains the extensive additions made by the author to the fourth German edition published in 1901, and has been thoroughly revised throughout. The author's additions are considerable, occupying a space of more than seventy pages, the principal sections being those discussing Hertz's *Mechanics* and the concepts of mass, inertia, absolute

motion, etc. These elucidative remarks of the author will be found highly valuable by students of the philosophy and history of science, while the references to the recent literature in this department, which is very rich, will also be found helpful. The price of the book, which in the German is quite expensive, has been reduced, and the publishers have taken every measure to render the work attractive and accessible to the public for which it is intended.

VORTRÄGE ÜBER DESCENDENZTHEORIE GEHALTEN AN DER UNIVERSITÄT ZU FREIBURG IM BREISGAU. Von *August Weismann*. Mit 3 farbigen Tafeln und 131 Textfiguren. Jena: Verlag von Gustav Fischer. 1902. Pages, Erster Band, xii, 456. Zweiter Band, vi, 462.

After a long silence and after the smoke of the great controversy which ten years ago or more hung over the battle-fields of biology has entirely vanished, the name of Weismann again appears upon the literary horizon. There is a note of sadness in the present two volumes of lectures on the *Theory of Descent*. Its author remarks that "when a life which has been full of the joy that labor brings is nearing its end, a desire inevitably arises to put into symmetrical and harmonious form the main results of one's achievements, as a bequest to the generations to come." A painful affection of the eyes has greatly hindered his labors, and he feels uncertain whether time or strength will long be left him for the further improvement of his biological system. These have been the motives which have led him to the present publication. It is hoped that his apprehensions will not be fulfilled.

The work is a collection of lectures on general biology which the author delivered before a mixed audience of students of medicine and natural sciences. Weismann still stoutly maintains that, despite the opposition which they have encountered, his biological views of descent and heredity are essentially and fundamentally correct,—notably his assumption of the "determining" units of life (determinants) and their relation to the ids. Closely connected with the theory of determinants stands his theory of Germinal Selection (expounded some years ago in the pages of *The Monist*), without which, he claims, the great thought of the controlling influence of natural selection in the transmutation of living forms with the rejection of the unfit and the selection of the fit, would remain a torso, a tree without roots. The application of this principle of selection to each and every single element of the various categories of vital units, is, he states, the very gist and kernel of his biological views. It is the dominating thought of the lectures, that in which their significance centers. "It will endure," says Weismann, "even though everything else the work contains shall perish."

Whatever may be the opinion of critics on this point, none can gainsay that the picture of modern biological research, which the great inquirer has here unrolled, is one of great fascination and replete with instruction.

GRUNDRISS EINER GESCHICHTE DER NATURWISSENSCHAFTEN. Zugleich eine Einführung in das Studium der grundlegenden naturwissenschaftlichen Litteratur. Von *Dr. Friedrich Dannemann*. I. Band. Erläuterte Abschnitte aus den Werken hervorragender Naturforscher aller Völker und Zeiten. Zweite Auflage. Mit 57 Abbildungen zum grössten Teil in Wiedergabe nach den Originalwerken und einer Spektraltafel. Leipzig: Verlag von Wilhelm Engelmann. 1902. Pages, xiv, 422. Price, bound, 9 Marks.

The fact that the first edition of Dr. Dannemann's *Elementary History of the Natural Sciences* has been exhausted within a space of five years is evidence of the great educational value of the volume. To those who are unacquainted with the nature of the work, we may say that it consists of two volumes, the first of which, now in its second edition, gives specimen classical investigations from all the sciences, in the words of the original discoverers and authors; for example, selections from Archimedes, Copernicus, Galileo, Kepler, Pascal, Guericke, Newton, Huygens, Celsius, Kant, Franklin, Cuvier, Faraday, Liebig, Darwin, Schönbein, and Hertz. The little articles are accompanied by neat fac-simile reproductions of the cuts and pictures of the originals from which the extracts were taken. Dr. Dannemann's work, with its companion volume, which is a connected historical narrative of scientific discovery, cannot be too highly recommended.

DICTIONARY OF PHILOSOPHY AND PSYCHOLOGY. Including many of the principal conceptions of Ethics, Logic, Æsthetics Philosophy of Religion, Mental Pathology, Anthropology, Biology, Neurology, Physiology, Economics, Political and Social Philosophy, Philology, Physical Science, and Education, and giving a Terminology in English, French, German, and Italian. Written by many hands and edited by *James Mark Baldwin*, Ph. D. (Princeton), Hon. D. Sc. (Oxon.), Hon. LL. D. (Glasgow), Stuart Professor in Princeton University, with the co-operation and assistance of an international board of consulting editors. In three volumes with illustrations and extensive bibliographies. Vol. II. New York: The Macmillan Company. London: Macmillan & Co., Ltd. 1902. Pages, xii, 892. Price, $5.00 net.

The text of Professor Baldwin's *Dictionary of Philosophy and Psychology* is completed with the present volume, which contains helpful general indices of Greek, Latin, German, French, and Italian philosophical terms. The editor, in thanking correspondents and reviewers for bringing minor errors in Vol. I. to his notice, remarks that the book has been misjudged on two points only. His reviewers fell heavily upon the bibliographical portions of the *Dictionary*, and the editor parries their onslaughts with the remark that the treatment of biography was not "more than the proverbial 'part of a loaf'; it was a question, indeed, of part of a loaf or no bread." He also repudiates the insinuation that Greek and Latin philosophy have been slighted. He has nothing to say about the inclusion of a hodgepodge of irrelevant theological, juridical, and other terms. We opine that in this

case, continuing the baker's simile, it was a question of pumpernickel or no pumpernickel.

The present volume contains some articles of considerable length, like those on the "Nervous System," "Syllogism," and "Vision." We gave in *The Monist* for April, 1902, a rather lengthy general review of the character and tendency of Professor Baldwin's *Dictionary*, and it now remains for us merely to emphasise again its general usefulness and value as a work of reference, which it certainly possesses in spite of the many inconsistencies in the plan of its construction. A third volume of bibliography is still to appear, and it is to be hoped that this will prove an indispensable boon and guide to students, amidst the mountainous literature which has grown up around philosophical and psychological subjects.

GRUNDZÜGE DER PHYSIOLOGISCHEN PSYCHOLOGIE. Von *Wilhelm Wundt*, Professor an der Universität zu Leipzig. Fünfte völlig umgearbeitete Auflage. Zweiter Band. Mit 153 Abbildungen im Text. Leipzig: Verlag von Wilhelm Engelmann. 1902. Pages, viii, 686. Price, bound, 16 Marks.

OUTLINES OF PSYCHOLOGY. By *Wilhelm Wundt*. Translated with the Coöperation of the Author by Charles Hubbard Judd, Ph. D., Instructor of Psychology, Yale University. Leipzig: Wilhelm Engelmann. London Agents: Williams and Norgate. New York Agent: Gustav E. Stechert. 1902. Pp., xxii, 390. Price, $2.00 net.

We are glad to record the appearance of the second volume of the fifth and fully revised edition of Professor Wundt's *Elements of Physiological Psychology*. This celebrated production has occupied a unique and commanding position in experimental psychology, and was for years the sole work from which students drew their knowledge and inspiration. The text has been subjected to thoroughgoing alterations, and the work thus seems destined to exert a powerful influence for a long time to come.

We also welcome the issuance of a second revised English edition of Wundt's smaller *Outlines of Psychology*. It has been translated from the German by Dr. Charles Hubbard Judd, instructor of psychology in Yale University. This edition has been made to include all the additions the author incorporated in the fourth German edition, which are considerable. The phraseology of the English translation has also undergone alteration, and the helpfulness of this exposition of the great psychologist's theories has thus been enhanced in many respects.